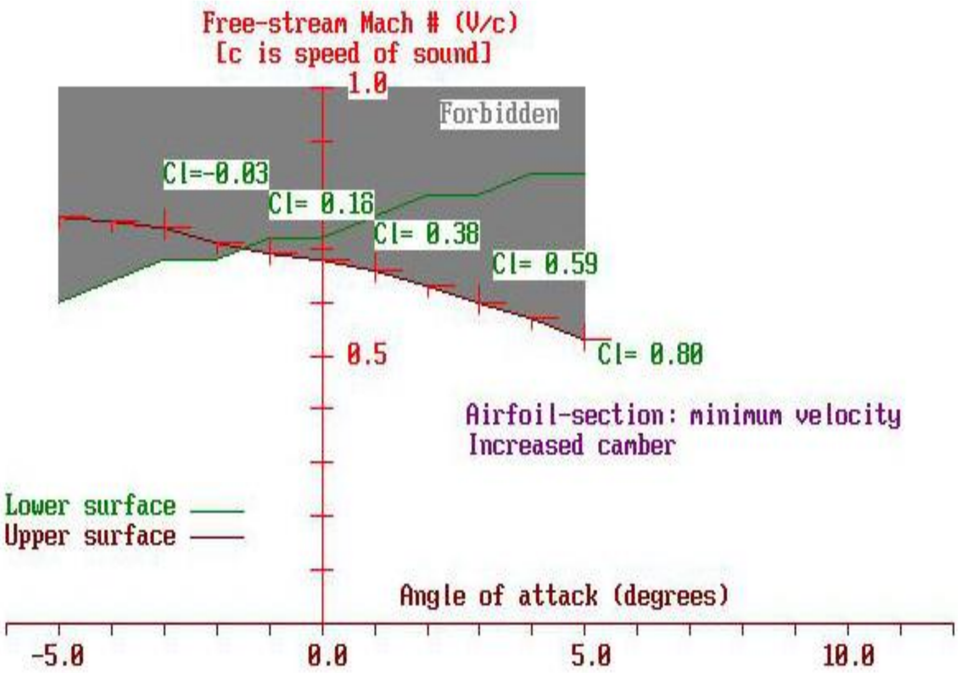
Critical Mach Number by Joe Supercool



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Part 1: M = free-stream Mach number
m = metre
s = second

Part 3: p = pressure
t = temperature
ρ = density
c_p = specific heat at constant pressure (ability to store heat, a known value)
c_v = specific heat at constant volume (ability to store heat, also a known value)
c = local speed of sound
$\gamma = c_p / c_v$
V = local air-speed over airfoil
p_o = pressure at a remote point in the free stream
ρ_o = air density at a remote point in the free stream

Part 6: M = free-stream Mach number
V = local air-speed over airfoil
c = local speed of sound
c* = critical speed of sound
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Critical Mach number: provenance.

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Critical Mach # Part 0: **Introduction**

The selection of airfoils for use in a propeller design is strongly affected by the speed of the propeller tip through the air. The issue is complicated by some perverse behaviour of the air itself. To reach an understanding of this behaviour, one needs a prepared mind.

For this reason, the following discourse has been divided up into 12 parts. Each part is intended to be complete internally; however, the preceding parts should be read first. This is the best way to "prepare the mind" for the current topic.

First a disclaimer. This article has not been peer reviewed, or even edited for mistakes. (Actually, the text has been edited, with many subsequent revisions, by my good friend and aerodynamic muse of many years standing, Peter Somers). The (remaining) errors present are made by me. I am proud of them. It has taken me three score years and ten less one to become this arrogant; proud I am of my mistakes as well as my few successes. Falls coming after pride are a badge of honour to me; now to work.

Part 0 you are now reading, so persist a little longer, then you will have the gist and scope of the work ahead.

Part 1 waffles on about the nature of losses in propeller tips due to their high speed through the air.

Part 2 is a brief rant about how air is made up of tiny little pieces, of which there are many. The discussion is necessary because we are going to ignore these little pieces altogether.

Part 3 introduces some big words like "thermodynamics", "macroscopic", "enthalpy" and others. We can get along quite well without these words, so we shall. They are included purely to cheer up those masochists who were so bent of mind as to study the evil subject of "thermodynamics".

Part 4 is about "fluid dynamics", a topic we have addressed previously on this website. Don't refer back to these previous articles. They are full of weird maths that we don't need here. After reading a pile of material on shock waves written by professors, I have decided that maths is the refuge of the obfuscator.

Part 5 is the good stuff about airfoil design. I don't understand much about airfoil design: luckily I don't have to. Chaps like Jones (R.T.) and Joukowsky (N.) have already done it, so all I needed to do was borrow their efforts.

Part 6 gets into Mach number, around which the whole world of aerodynamics revolves. Really, not kidding. To get an understanding of how shock waves form and behave, I have read quite a number of texts so that I could cherry-pick the equations that looked useful to me. No sooner had I found one and noted it down, than on turning the page it had been changed, the dreaded symbol M appearing out of nowhere! "C" is for "Cat", and "M" is for "Mach". Remember this; your life depends on it.

Part 7 reviews the nature of shock waves, but does not say anything about how they arise. Or more importantly, it says nothing about how we may predict their occurrence. That makes it all seem rather a waste of time, but again, only the prepared mind will follow the following parts. Things get a bit weird from here on in. You have been warned.

Part 8 is really abstract. It is about “similitude”; even the authors who brave this topic seem to be confused by it. Unfortunately, similitude is central to predicting the onset of shock waves: i.e., computing the critical Mach number. We are going to use data derived from assumptions of incompressible flow to a real world situation of compressible flow. This usage depends on kinematic and dynamic similarities being valid for the transformation from incompressible to compressible flow.

Part 9 invokes the Prandtl-Glauert scaling rule. We have hammered this rule previously on this website. This time we look more closely at where the rule came from, the approximations in its formulation: indeed, whether it is in fact valid in the real world. Then we give the rule even more relevance to our airfoil designs.

Part 10. We here discuss the nature of shock wave formation.

Part 11, we look at ways to prevent shock wave formation on our propeller airfoils in the transonic speed range. Well, prevent is the wrong word; avoid and delay are better. Sort of aerodynamic procrastination. This will involve changing the contours of the “parent” airfoil, then setting the new “daughter” airfoil to the “correct” angle of attack.

Part 12, we raise the subject of aerodynamic drag, which up till now we have comprehensively ignored.

Thank you for reading through this introduction. Your time is valuable to us. We will get back to you sometime.

Critical Mach # Part 1: **Propeller Tip Problems**

The ability of a propeller to absorb engine power depends on the diameter of the prop; also on the rate at which the prop is spinning. Both of these factors result in airflow velocity over the propeller tip which is in excess of the speed of the airplane. Thus the flow regime over the propeller may turn out to be more problematic than that over the airframe.

For low powered airplanes in the early days of flight, the problems were simply not encountered. But in the 1920's, propeller efficiency began to suffer due to the very thick airfoils needed for strength in wooden propellers. Forged aluminium propellers, developed by Reed (a dentist) in the USA, could be made with airfoils much thinner than their wooden counterparts. These new propellers showed a clear improvement in performance: despite problems with metal fatigue, these propellers became widely adopted for use in high speed airplanes.

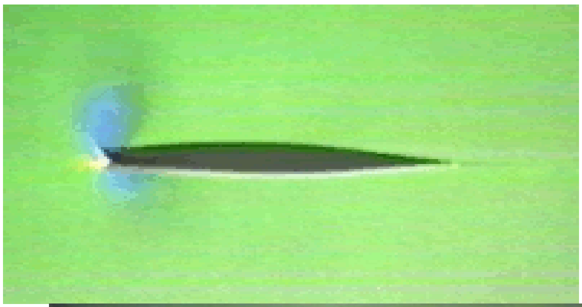
Not that the take up of metal propellers was rapid. Even in the late 1930's the British Spitfire was still using wooden, fixed pitch propellers. Indeed, even with introduction of variable pitch propeller hubs, the British were still using reinforced wooden propellers. Not that wood has an unlimited fatigue life. The fatigue failures in metal props usually resulted in loss of the propeller tip, due to flexing of the blade.

However, we are here concerned with a different type of problem, not concerned with the structure of the propeller; rather, the ability of the propeller airfoils to generate lift. The problems first became apparent near the prop tips. It was found that propeller airfoil sections moving at speeds just over half the speed of sound were generating poor lift with a great deal of drag. The sections used were typically of the form of Clark-Y and RAF-6 (Ref. 1)

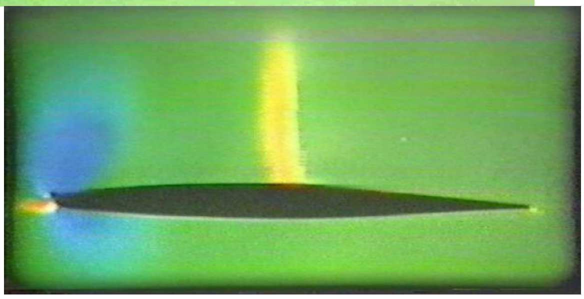
The latter airfoil is essentially Clark-Y with the nose camber removed: i.e., what we commonly refer to as a "flat-bottom" section. At low speeds these sections perform well, also being easy to manufacture. Accordingly, these sections were carefully preserved all the way out to the tips.

The use of these sections all the way to the tips was essentially a disaster. Above tip speeds of 250 m/s (metres per second), these airfoils behave very badly. Not only do they produce a great deal of noise, but also very little thrust. Indeed, it is quite possible at 300 m/s for these sections to produce negative lift (Ref. 2). That is to say, reverse thrust. Not a good look!

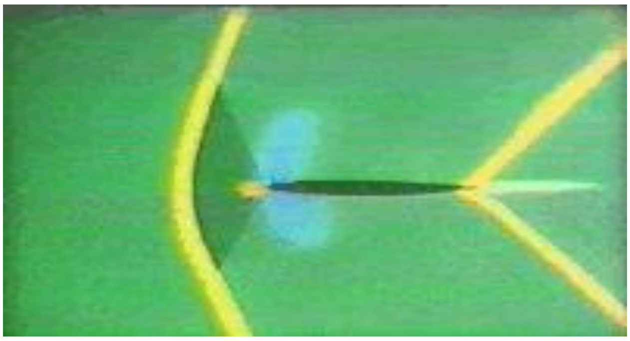
The cause of the problem related to 2 effects: compressibility of air and the formation of shock waves. These phenomena can be divided up into flow regimes as follows. Defining Mach number “M” as the free-stream speed of the air divided by the speed of sound in the free-stream, we have:



$0 < M < .7$ The subsonic region, none of the local flow is supersonic.
The blue area shows reduced pressure, yellow is increased pressure at the stagnation point.



$0.7 < M < 1.2$ is the transonic region, where at least some of the local flow is supersonic.
The shock wave is visible due to the high pressure gradient across the transition.



$1.2 < M < \sqrt{6}$ is the supersonic region, all the flow being faster than sound.
A bow-wave stands off from the leading edge, immediately followed by a subsonic region (in dark green)

The subsonic region is marked by the air becoming more compressed as flow speed increases over the airfoil. The effect of this compressibility is to change the apparent “shape” of the airfoil. Indeed, the airfoil starts to behave as though both its thickness and camber have been increased: indeed, the angle of attack also appears to increase. For these reasons, the propeller airfoil needs to be thinned, its camber reduced, while blade angle of attack should also be reduced.

The transonic region is far more compromised. The flow over the airfoil becomes disrupted by the formation of a shock wave at the point on the airfoil surface where the flow speed first exceeds the local speed of sound. As the shock wave increases in intensity with more speed, lift falls, drag rises sharply and the noise is just a wonder. Indeed, it has been said that propellers have the greater power to produce continuous, intense noise than any other source.

As for the supersonic region, well we really don’t want to go there!

Returning to the transonic region, we should see what factors reduce the shock wave. There is no way of deleting the wave: however, it can be reduced in intensity and the shock formation delayed by cunning changes in the shape of the airfoil. The most immediate change is to thin the airfoil, thereby reducing the camber as well. The curvature of the section being reduced, the local flow speed is also reduced, delaying the point where the local speed exceeds the local speed of sound. This method is called "raising the critical Mach number".

During the 1950's, the introduction of swept-back wings was found to be effective in raising the critical Mach number. Then in the early 1970's the notion of "supercritical" airfoils gained currency. By effectively "flattening off" the upper surface, increasing the negative camber of the lower surface, adding a slight downward "cusp" to the trailing edge and rounding the leading edge, the critical Mach number was again raised (Ref. 3). This method was so effective that the sweep-back angle could be reduced, while the wing could also be thickened to gain bending strength.

We will examine this method in detail in Part 11. However, your brain is not yet ready for this, so move on to Part 2.

References:

1. Wilbur C. Nelson (1944). "Airplane Propeller Principles". John Wiley and Sons.
2. Ira H. Abbot and Albert E. von Doenhoff (1959). "Theory of Wing Sections", p275, Fig. 163. Dover Publications Inc. SBN 486-60586-8
3. Ray Whitford (1987). "Design for Air Combat", p27-34. Janes. ISBN 0 7106 0426 2

Critical Mach # Part 2: **Molecular nature of air**

Aerodynamics is a great disappointment to me. After I finished studying the kinetic theory of perfect gases, I figured I had the world at my feet. Here was a beautiful Newtonian explanation of temperature and pressure; a mechanism for explaining the forces acting on a surface in air. Momentum exchange, just the ticket. Think billiard balls. In case you have for forgotten, here are Newtons Laws of Motion, as drummed unforgettably into my infertile puerile teenage mind by my physics teacher, Mr Steanes.

1. Every body continues in a state of rest or uniform motion in a straight line unless compelled to change that state by some external force.
2. The time rate of change of momentum is proportional to the impressed force, and takes place in the direction of that force.
3. To every action, there is an equal and opposite reaction.

Pressure is just the result of lots of little molecules jumping around, occasionally bumping into a surface, such as an airfoil. Result? Lift. Easy.

Temperature is just that the little molecules race around a lot faster, banging into things a lot harder. Also easy.

Now we jump into aerodynamics, with Bernoulli's equation. This famous equation tells us that the pressure in faster moving air is lower than in slower moving air. Interesting, I always thought that wind is more likely to blow me over than pick me up. So I looked more closely at the equation and its derivation. Do I find little molecules jumping around? Is there any mention of my beloved kinetic theory of gases? Where is the momentum exchange?

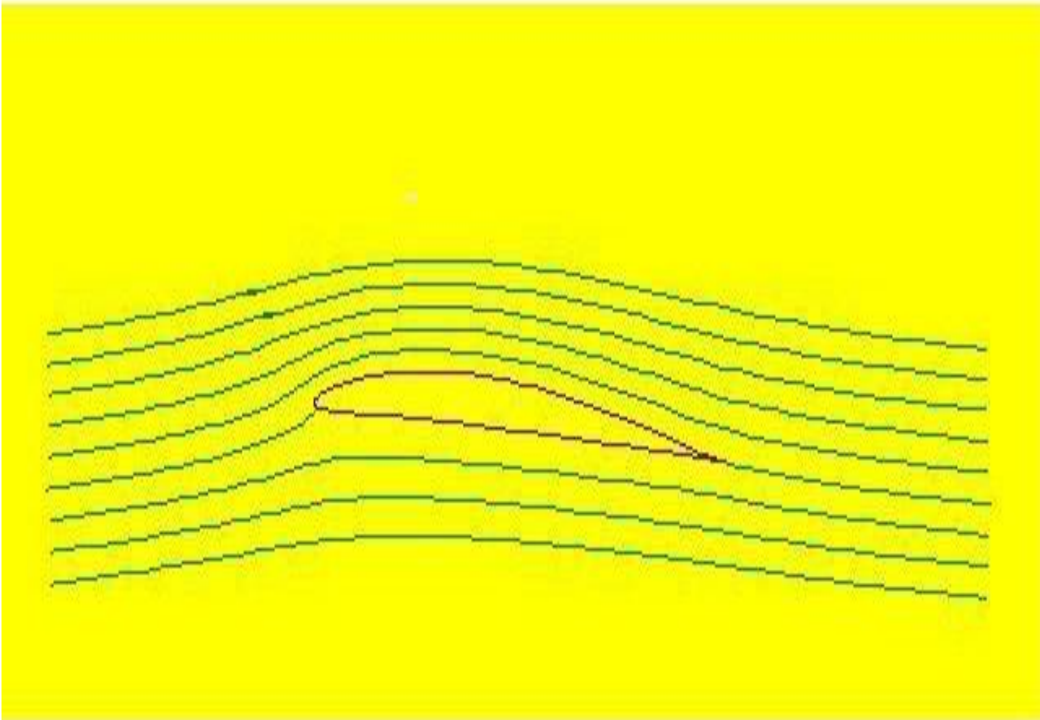
I'm not saying Bernoulli is wrong. What I am saying, having read the equation very, very carefully, is that I am none the wiser. Why does the pressure in moving air fall lower than in stationary air? Where are the little molecules? More tellingly, why can't I find a derivation of Bernoulli's equation based on the kinetic theory of gases? Who has been asleep at the wheel?

Even more worryingly, how am I supposed to understand shockwave formation if the little molecules don't get a mention? I am not alone in all this. I recently read an article in an amateur ultra-light aircraft journal that counted up no less than 12 different explanations of how an airplane wing generates lift. All different, not all wrong, but all confused.

Before my time on this planet expires, my ambition is to derive Bernoulli's equation directly from the motion of gas molecules. Indeed, I have already started. But meantime, I have a problem. How do I explain aerodynamic forces without using Bernoulli?

I cannot. So dear reader, YOU must take this on face value.

FAST MOVING AIR HAS LOWER PRESSURE THAN SLOWER MOVING AIR



In the plot of streamlines above, the separation of the adjacent streamlines is a guide to the local airspeed and local pressure. On the upper surface, the streamlines are close to each other, indicating increased speed and lower pressure: this surface is called the “suction” side. On the lower surface, the streamlines are further apart, indicating lower local air speed and higher pressure: this surface is called the “pressure” side.

Well not quite. The opposite is true for air moving faster than the speed of sound!

Time to read on. I did warn you that this Part 2 was a rant. Well here is another warning. Part 3 is next. Horror of horrors, there are equations. Yes, thermodynamics!

Critical Mach # Part 3: **Thermodynamics of air**

Don't panic. Supercool is here to guide you through the intricacies of thermodynamics. You can trust Supercool on this because air itself is so nice to work with. It really is hard to go wrong with air: primarily because the little air molecules behave like a perfect gas. Unfortunately, this is our last mention of the little air molecules. When we look at the behaviour of molecules, we are taking a MICROSCOPIC view of the world. But there are such vast numbers of these little chaps we tend to lump them all together and call them a fluid.

Such a way of regarding the gas as a fluid is called the MACROSCOPIC view. Thermodynamics is the macroscopic view of the fluid. The microscopic view of the fluid is called Statistical Mechanics: we will steer well clear of this latter topic!

If we are to get anywhere in understanding shock wave formation, we need to first appreciate the behaviour of air as a fluid. Firstly, the fluid has no structure. No structure, just properties. The properties are temperature, pressure, density, capacity to store energy, and heat transfer. The latter we can discard right away. Air is a lousy substance for transferring heat by conduction. We can ignore any processes that involve loss of heat by conduction. There is a word for this: it is "adiabatic" (Ref. 1).

Moving right along, we need to consider capacity to store energy. The reality is that there is pressure in air, mainly from the weight of the air above, but also due to heat. This energy can be transformed into other types of energy, such as the energy of motion called kinetic energy. There is a word for the combined energies of kinetic and pressure processes: the word is "enthalpy" (Ref. 2). Nice to meet you, hope we never meet again. If my memory serves me well, there is a word for the energy stored as heat: it is "internal energy" (Ref. 3).

This is all getting a bit heavy. So here is a list of items which will be useful to us. (Ref. 4)

p = pressure
t = temperature
ρ = density
c_p = specific heat at constant pressure (ability to store heat, a known value)
c_v = specific heat at constant volume (ability to store heat, also a known value)

Two more useful quantities are:

c = speed of sound
$\gamma = c_p / c_v$

The quantities p, t, and ρ are connected to each other by the perfect gas law. The quantities p, ρ and γ are connected because the processes in consideration are adiabatic. Finally, the speed of sound c is connected to p, ρ and γ.

So what? Well, what physicists do now is throw all these variables into a bag, shake vigorously and see what comes out. Well, it turns out that given any one item, say p, then all the others are known right away (Ref.5). This is almost too good to be true. When I said that air was nice to work with, this is exactly what I meant!

All very well, but with another shake of the bag, we manage to combine Bernoulli's equation with the adiabatic condition to get the Saint-Venant and Wantzel equation, which I offer below (Ref. 6):

$$V^2 = (2\gamma p_o) / ((\gamma-1)\rho_o) [1-(p/p_o)^{(\gamma-1)/\gamma)}]$$

The subscript in p_o and ρ_o places the pressure and density values at a remote point in the free-stream.

The importance of this equation is that it links the velocity (V) of flow of the gas with the pressure in the gas. Thus if we know the speed of the fluid over an airfoil, we also know the pressure acting on the surface of the airfoil at that point. This is an extremely useful result, as we can use the methods of fluid dynamics to find the flow speed over the airfoil at any point. Indeed, we can get the pressure anywhere in the fluid as the methods of fluid dynamics also reveal the streamlines!

But we are not done yet. Recall that we threw the speed of sound c into that bag before we shuffled it: this is where the story really starts.

Firstly, it turns out that the speed of sound is not constant. The speed of sound varies according to the absolute temperature, but more interestingly from our fluid flow point of view, the speed of sound depends on the velocity of the fluid! Thus if we follow along a streamline, we will find that the speed of sound can vary along the streamline; even at a place remote from the airfoil. Understanding and appreciating this fact is the key to understanding the formative process of shock waves. The equation linking the speed of sound c with the velocity of flow V is Ref.7):

$$c^2 = (\gamma-1)[V^2_{max} - V^2]/2$$

where

$$V^2_{max} = (2\gamma/(\gamma-1))\, p_o/\rho_o$$

A word of caution about using these equations: there are 2 conditions which must be met. The flow stream lines must follow a "smooth" trajectory; the velocity V of the gas flow must be less than the speed of sound c. This latter point will become critical when we turn our attention to shock wave formation. Remember this: sound is transmitted through the air, but is not carried along by the air. What does that imply? If the air is blowing toward the observer, then an increase is sound pitch (frequency) is noted, while if the air is blowing away from the observer, the pitch decreases.

Now a warning. The variables p, t and ρ are linked by an equation, called the equation of state (Ref. 8). The truth of this equation requires that the above variables are changed only slowly in any given process. We are going to examine a process, that of shock wave formation, in which the changes of state are very rapid: care is required.

This is enough on the thermodynamics. We have here all the equations we need to understand the mechanism of shock wave production. Part 4 moves on to a look at what Fluid Dynamics can do for us.

References:

1. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p2. Mc Graw-Hill.
2. P. S. Barna (1957). “Fluid Mechanics for Engineers”, p238. Butterworths.
3. P. S. Barna (1957). “Fluid Mechanics for Engineers”, p239. Butterworths.
4. E. L. Houghton and A. E. Brock (1960). “Aerodynamics for Engineering Students”, p8. Edward Arnold.
5. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p2. Mc Graw-Hill.
6. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p13. Mc Graw-Hill.
7. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p17. Mc Graw-Hill.
8. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p1. Mc Graw-Hill.

Critical Mach # Part 4: **Fluid Dynamics**

The time has come when we must get some information on the actual flow of air over an airfoil. The mathematical process covered by the term "Fluid Dynamics" has the power to provide this information. The method takes advantage of the "nice" properties of air by assuming the air has no viscosity (is inviscid) while also being incompressible (density is fixed). The first assumption of zero viscosity is nearly true. The second, of incompressible material, is truly and obviously false. Air is very easy to compress. Just clap your hands and hear the report: that is compressed air setting up a sound wave.

Despite this, the airflow over a smoothly shaped airfoil does not do much compressing, at least for airspeeds below 150 m/s. We shall assume incompressible flow as an initial starting point. Later, in Part 9, we will show how data based on this assumption can be adapted to give valid results for compressible flow.

As the free-stream encounters the leading edge upper surface of the airfoil, the air is speeded up by being deflected over the surface. As the air passes the airfoil high-point, the speed starts to drop back, slowing down again to the free-stream speed. On the under-surface, quite a different event occurs. The air passing under the airfoil slows down, quite counter-intuitively. Words can't do this event justice, so refer to Figure 1 below.

Figure 1: Streamlines for incompressible flow computed using Joukowsky-Jones transform.

Observe varying widths across the streamlines. These widths indicate varying airspeed along the streamlines.

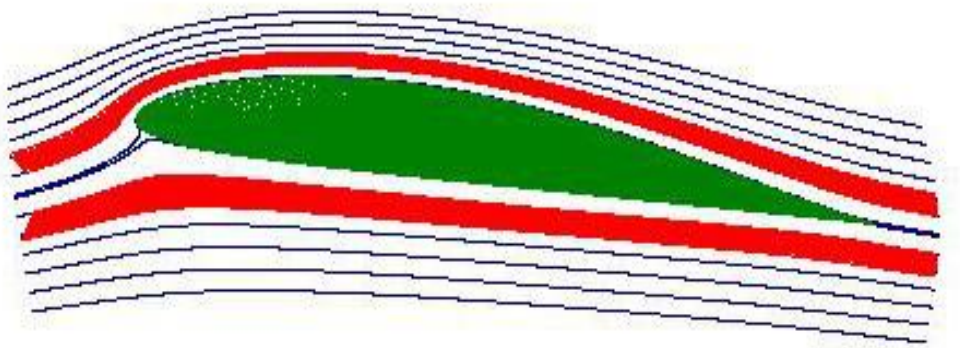


Figure 2: Velocity along streamlines for incompressible flow computed using Joukowsky-Jones transform. Observe upper surface velocity increased, lower decreased.

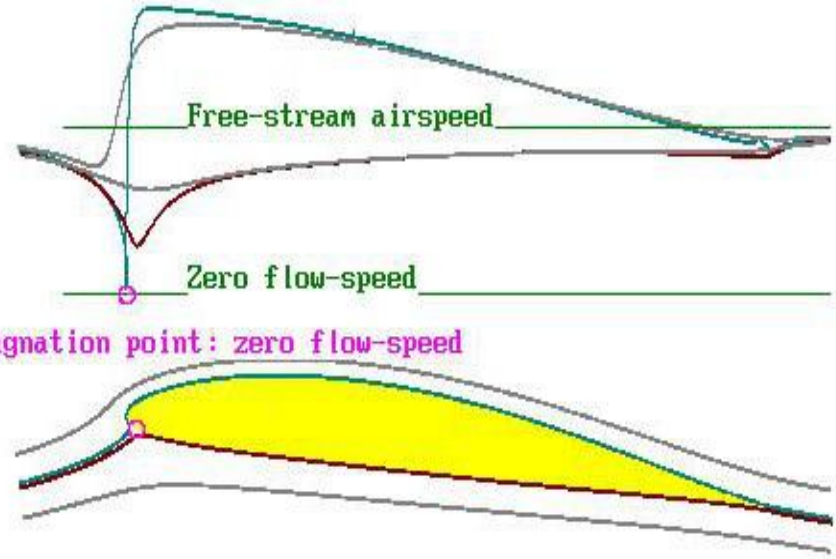


Figure 3: Pressure along streamlines for incompressible flow computed using Joukowsky-Jones transform. Observe upper surface pressure decreased, lower increased.

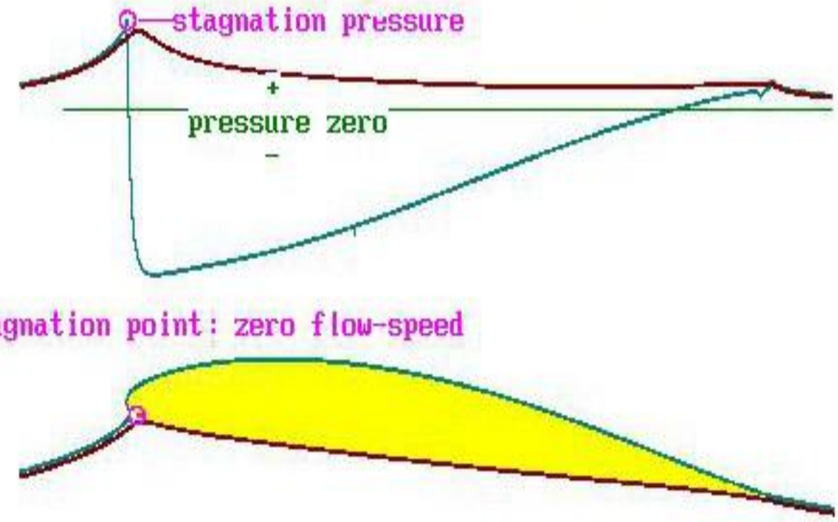
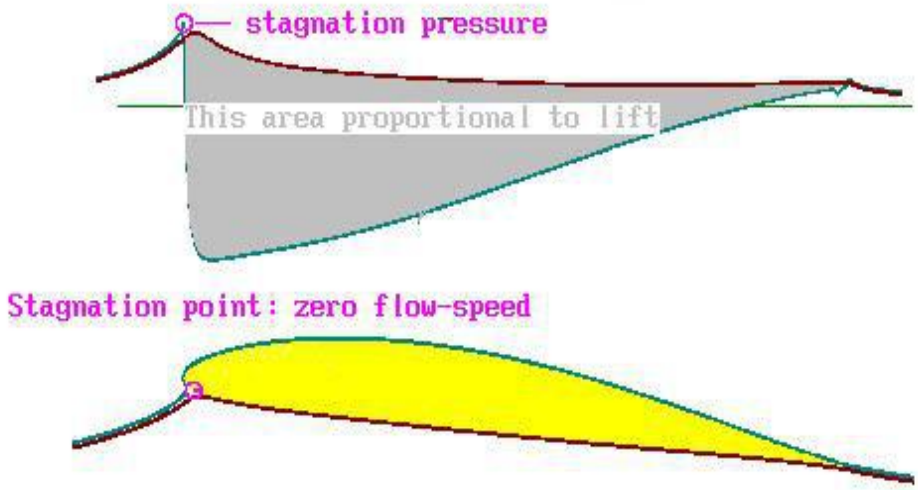


Figure 4: Pressure summed between the upper and lower surfaces gives the lift force.



The stream-lines here tell an amazing story. Not only do the streamlines on the upper surface go faster than the free-stream, but the stream-lines on the under-surface go slower than the free-stream (Ref. 1). Not only that, but the streamlines do not part at the leading edge, but rather at a point under and rearward of the leading edge (Figure 2)!

This was all too much for Hiram Maxim (Ref. 2). Here is what he had to say about fluid dynamic calculations in 1908.

"Mathematics of the higher order expressed in elaborate formulae do very well in communications between college professors – that is, if they happen to be agreed. When, however, these calculations are so intricate as to require a clever mathematician a whole day to study out the meaning of a single page, and if when the riddle is solved, we find that these calculations are based on a fallacy, and the results in conflict with facts, it becomes quite evident to the experimenter that they are of little value".

Amen to that, I couldn't agree more. But Maxim became more and more exasperated. Here are more of his views on the maths of fluid mechanics (Ref. 3).

"During the last few years, a considerable number of text-books have been published. These have for the most part been prepared by professional mathematicians, who have led themselves to believe that all problems connected with mundane life are susceptible of solution by the use of mathematical formulae, providing, of course, that the number of characters employed are numerous enough. When the Arabic alphabet used in the English language is not sufficient, they exhaust the Greek also, and it even appears that both of these have to be supplemented some-times by the use of Chinese characters. As this latter supply is unlimited, it is evidently a move in the right direction".

I suspect that Lanchester (Ref. 4) may have been the target mathematician here, athough Joukowsky was active at this time so I suspect they both got a serve. Nonetheless, it is Joukowsky who appears in all the text books these days (1950-2012).

Maxim has a case. In Professor Milne-Thomson's work on Theoretical Aerodynamics, there is a symbol page xvii with 70 (!) characters. Lots of Arabic and Greek, but no Chinese.

Referring back to Figures 1 and 2, we end this part on fluid dynamics with the following points derived from the mathematical theory (Ref. 5).

1. The path followed by a given particle of air over the airfoil is indicated by a stream-line.

2. The speed of this particle can vary along the stream-line.

3. The stream-line which matches the surface of the airfoil is called the dividing stream-line.

4. The stream-lines over the upper surface usually give rise to a reduced pressure. For this reason, the upper-surface is commonly called the "suction side".

5. The stream-lines over the lower surface usually give rise to an increased pressure. For this reason the lower surface is commonly called the "pressure side".

6. The speed of the stream-lines can be deduced from the separation of the stream-lines. Streamlines close together indicate increased speed; stream-lines far apart indicate reduced speed of the airflow.

7. There is a point near the leading edge where the dividing stream-line parts, to permit flow over the upper and lower surfaces. The flow speed at this point is zero, so the point is called the "stagnation point". The pressure at the stagnation point is called the "dynamic pressure", having the highest value of pressure anywhere on the airfoil.

8. The stagnation point does not in general lie at the geometric leading edge.

9. None of the air between two adjacent stream-lines can cross the stream-lines.

Now we move on to Part 5, where we meet a method for computing airfoil shapes and their stream-lines.

References:

1. Ray Whitford (1987). "Design for Air Combat", p10, Fig.3. Janes. ISBN 071060426
2. Hiram S. Maxim (1908). "Artificial And Natural Flight", Preface ix. MacMillan.
3. Hiram S. Maxim (1908). "Artificial And Natural Flight", p1. MacMillan.
4. F. W. Lanchester (1908). "Aerial Flight". Archibald Constable, London.
5. Joseph Katz and Allen Plotkin (2001). "Low-Speed Aerodynamics". Cambridge University Press.

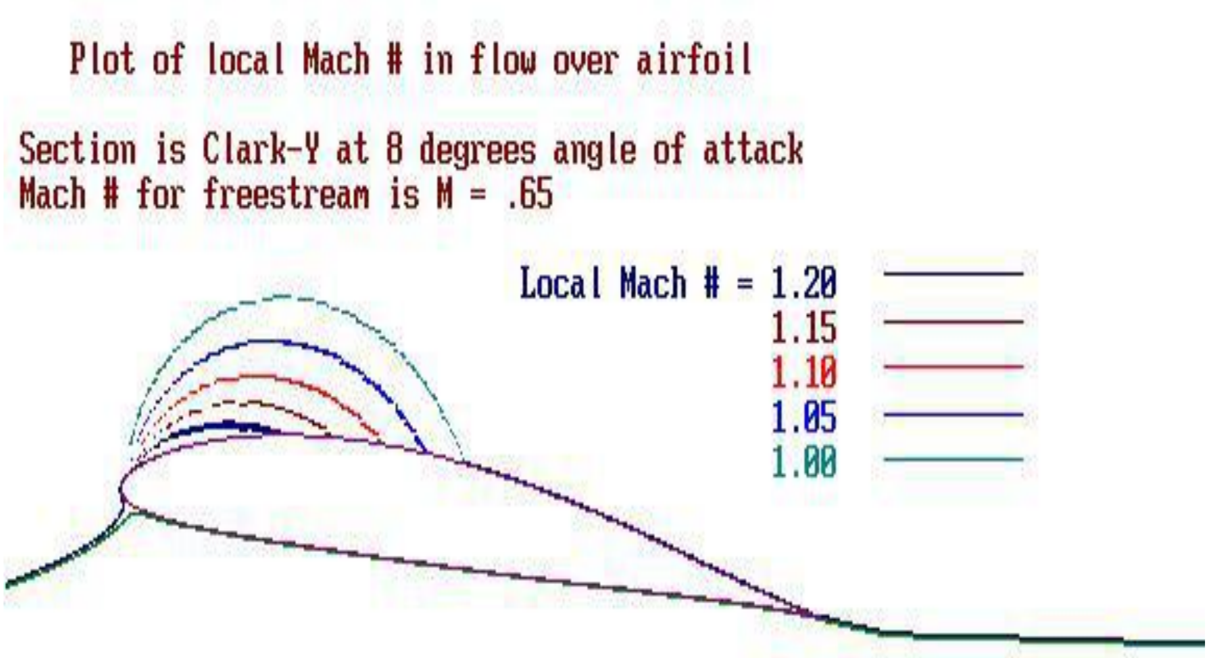
Critical Mach # Part 5: **Airfoil design code**

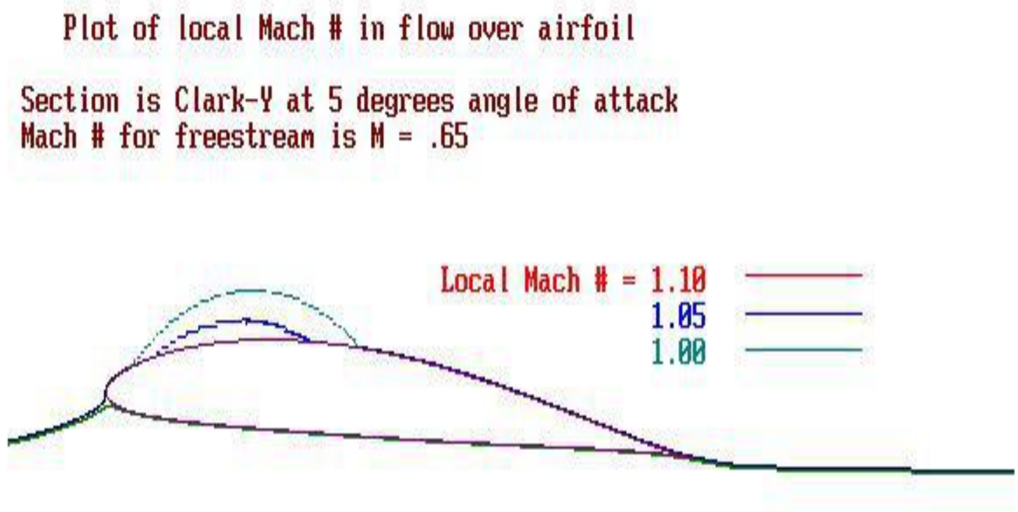
Back in 1988, Martin Hollmann published a book by R.T. Jones called “Modern Subsonic Aerodynamics” (Ref. 1). In this tract, Jones introduced an airfoil design method based on the Joukowski transform.

The design method required the specification of 5 parameters, all related to the geometry of the designed airfoil. The 5 parameters allow the designer to reproduce a good facsimile of virtually any useful airfoil shape. Included in an appendix was a BASIC program for executing and displaying the airfoil and its aerodynamic parameters.

The computer program was called the “Oshkosh Airfoil Program”, copyright 1984 to Rick McWilliams and adapted by Thomas Gans for the IBM-PC. I converted the code to Quick Basic, finding immediately that the code was of great value. Not only did it generate the airfoil shape, but also the associated stream-lines, lift-coefficient and zero-lift angle. The code also presented a plot of the chord-wise pressure distribution. Regrettably I could not extract from the code the velocity along the stream-lines, nor the pressure at the streamlines. I put this down to my own stupidity in being unable to read that part of the code.

At this time, I was worried that my proprietary propeller design code could not account in any way for shock wave formation. Since a number of classes (F2A, F3D) of model racing engines were running their propellers very close to tip speeds Mach 1.05, this was a real weakness. I already had compressibility covered by the Prandtl-Glauert scaling rule (Refs. 1, 2, 3, 4, 5, 6) for compressible flow, but nothing for flow speed in the supercritical region. The usual technique for delaying the shock wave formation is to reduce the airfoil camber: this is not much good if you don’t know what angle of attack to use, nor what lift coefficient is permissible.





The above plots show the local Mach number, assuming incompressible flow with no shock wave formation. The local speed of sound over the upper surface in both diagrams exceeds the speed of sound at the surface of the airfoil. The local Mach number is reduced by reducing the angle of attack. Regrettably, shock waves DO form so we have a problem!

Then it occurred to me that perhaps I could use the Jones-Joukowsky method for incompressible flow, by adapting it to compressible flow using Prandtl-Glauert scaling. By adjusting the airfoil shape using the 5 parameters, perhaps I could thwart or at least delay shock wave formation.

Accordingly, I hacked up the Oshkosh code to eliminate unused parts, selecting my own values for the stream function. In selecting close stream-line pairs, by their spacing I was able to generate the differentials of the potential function; which is to say, I obtained the velocities along the stream-lines. Application of the Saint-Venant and Wentzel formula then gave me the pressure along the stream-lines.

At this point, I had all the incompressible flow data necessary to pursue the critical Mach number problem.

In Part 6 we look at how the Mach number wormed its way into the equations for high speed compressible flow.

References:

1. Robert T. Jones (1988). “Modern Subsonic Aerodynamics”. Aircraft Designs Inc.
2. Joseph Katz and Allen Plotkin (2001). “Low-Speed Aerodynamics”, p227. Cambridge University Press.
3. F. W. Godsey and Lloyd A. Young (1949). “Gas turbines for Aircraft”, p23. McGraw Hill.
4. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p104-107. Mc Graw-Hill.

5. Ira H. Abbot and Albert E. von Doenhoff (1959). “Theory of Wing Sections”, p256. Dover Publications Inc. SBN 486-60586-8
6. L. M. Milne-Thomson (1958).“Theoretical Aerodynamics”, p276-278. MacMillan.

Critical Mach # Part 6: **Mach number**

In reading about jet fighters, the term Mach number is used to indicate the speed performance of the airplane. Incidentally, Ernst Mach lived from 1838 to 1916. The equation for Mach number M is given below, where V is the speed of the airplane and c the velocity of sound in ambient conditions (i.e., temperature at a given height, airspeed). Thus we write:

$$M = V/c$$

However, when we are considering flow over an airfoil, we have to be more specific. As noted in Part 2, the speed of sound is also a function of the local speed of the air over the airfoil. This speed varies along the stream-lines. Our simple formula for M (above) has therefore a variable "V" hidden within "c", thereby reducing the charm of the expression considerably!

There exists a value for V which is equal to the local speed of sound. Naturally, this implies that V is pretty fast, indeed, right up in supercritical range, where shock waves are likely to form. For this reason, we give V a symbol to represent the condition above. We write c as c* and V as V*: at the same time, we call V* the "critical" speed.

$$V^* = c^*$$

Now this is a very suspicious use of language. Is not the title of this article "Critical Mach number"? Could we be using the word "critical" a bit loosely? Or could V* refer to the critical Mach number which we are seeking? Indeed it could, so file this thought away in your mind until we get to Part 10.

Right now we want to drop M into our bag of formulae, giving the bag another good shaking. This time we will drop M* into the bag instead of M. We have (ref. 1):

$$M^* = V/c^*$$

where

$$c^* = [(\gamma-1)/(\gamma+1)]V_{max}$$

and then

$$p/p_o = [1-(\gamma-1)/(\gamma+1)\,M^{*2}]^{(\gamma/(\gamma-1))}.$$

Well that was rather a good shake! We now have the ratio of local pressure over the airfoil to free-stream pressure, written with the local Mach number included. This all seems rather miraculous, so we are entitled to wonder where all these fancy relationships came from!

Hearken back to Part 3, where we found that air is rather obliging when it comes to behaviour. In the thermodynamics of air, we found that to completely determine the state of the air, we need only to have a value for one of the macroscopic variables, say pressure. Then under the conditions that the flow over an airfoil is adiabatic (no heat transfer) and that Bernoulli's equation holds (smooth airfoil), then everything else is known (Ref. 2). All we have done to get to this equation is rattle all the previous equations around together. No magic at all.

So what do we need to take away from here? Well, we must be very careful to remember that the speed of sound varies along the streamlines, in a way that is dependent on M*. To put in a number, if the speed of sound in still air is 340 m/s, then the speed of sound in air moving at 340 m/s is only 313 m/s.

Further, there exists a value for the speed of sound, the so-called critical speed c*, where the speed of flow V is equal to c*.

We can expect trouble when V = c*

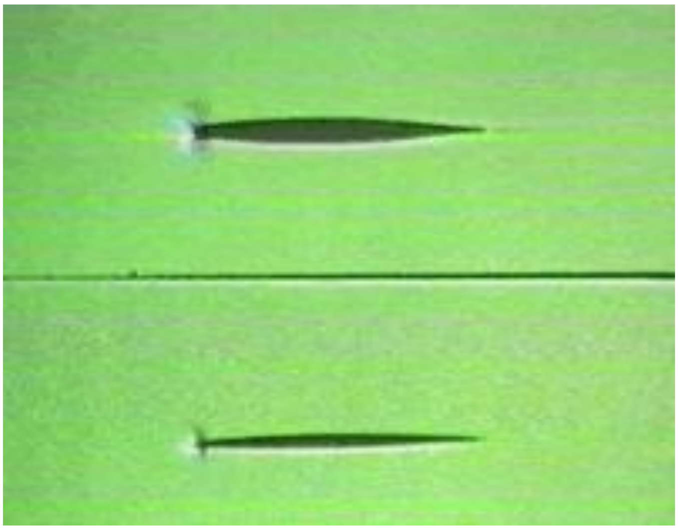
Now we have said a lot about something of which we have not shown anything. The time has come in Part 7 to have a look at some shock wave photos. A careful study of the photos in Part 7 will help prepare us for the difficult time ahead.

References:

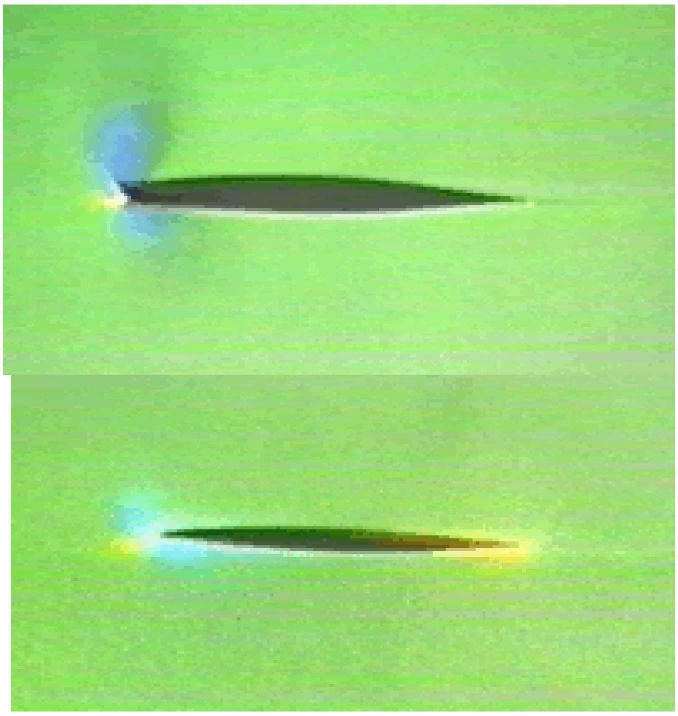
1. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p1-19. Mc Graw-Hill.
2. Edward R. C. Miles (1950). “Supersonic Aerodynamics”, p2. Mc Graw-Hill.

Critical Mach # Part 7: **Shock waves**

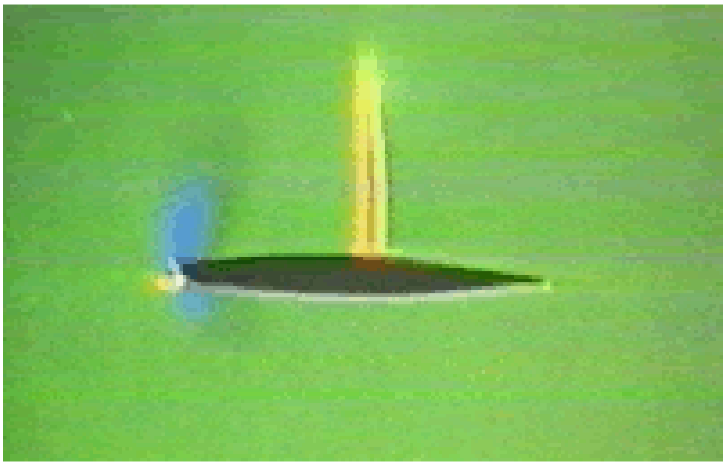
On the basis that a picture is worth a thousand words, the time has come to look at some wind-tunnel photos of airfoils in high speed flow. In looking at these pictures, you need to recall that we are not trying to produce a theory for these images here. We remain fixated in finding the conditions at which the shock waves start to form: that is, the critical Mach number.



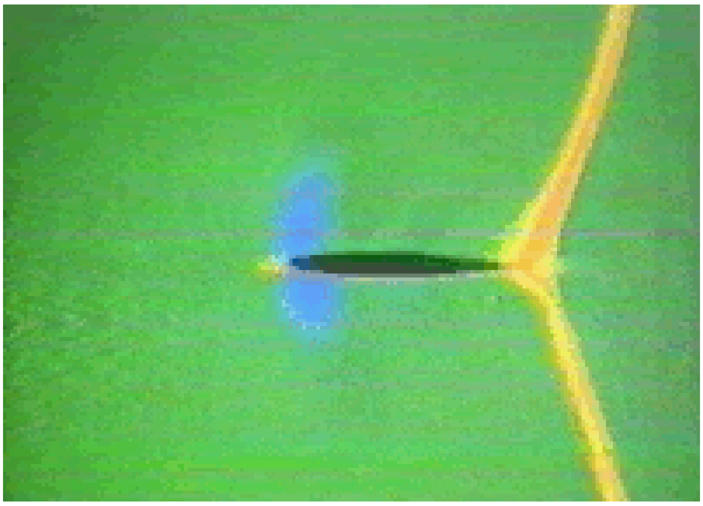
The two images at left show a thick airfoil and a thin airfoil at low speed. The blue area at the leading edge indicates reduced pressure. This is the region of “incompressible” flow.



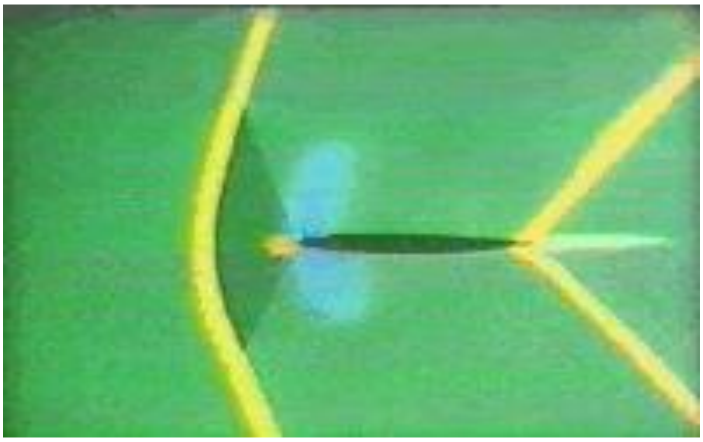
Same airfoils again, this time at higher airspeed with a slight increase in angle of attack. The blue areas of reduced pressure are greater, in proportion to the increased speed of flow over the airfoil



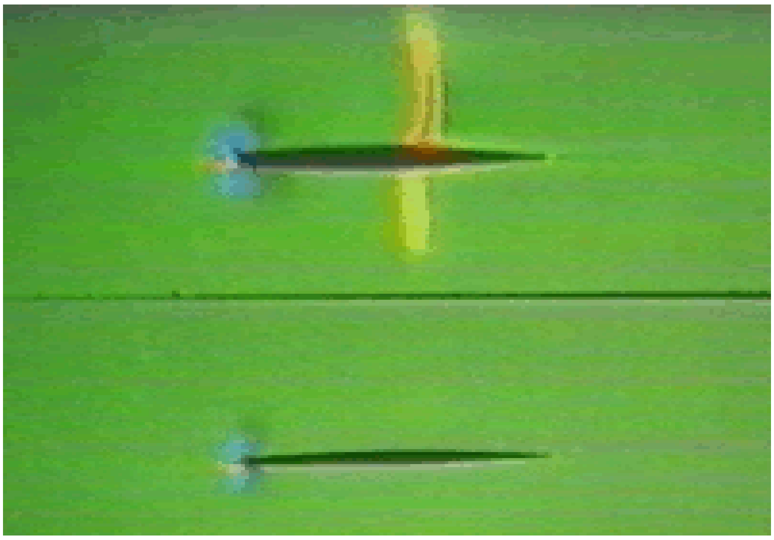
The image at left arises as the speed of flow in the wind tunnel is increased further. This time, the critical Mach number has been exceeded, with a strong shock wave formed on the upper surface.



Tunnel speed is at Mach 1. Shocks are present on both upper and lower surfaces. Both shocks have moved to the trailing edge



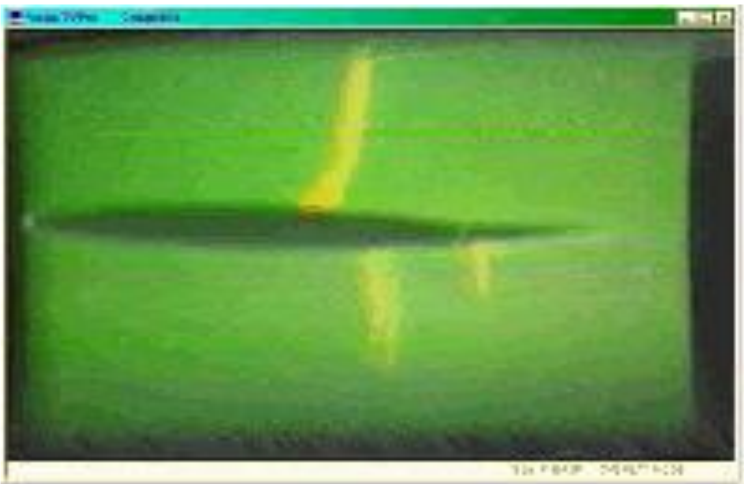
The tunnel speed is now supersonic. A bow wave (detached leading edge shock) has formed, behind which lies a subsonic area (dark green). At the leading edge, the high pressure at the stagnation point remains, as do the low pressure areas (in blue). Behind the leading edge, the flow speed has again increased to supersonic speed leading to further shocks at the trailing edge.



Finally, we return to the thick and thin airfoils, this time at yet higher speeds. Both airfoils are at the same speed of flow. However, the critical Mach number has been exceeded on the thick airfoil, with shocks appearing on both upper and lower surfaces. The local flow speed on the thin section remains below the critical Mach number, so no shock has yet arisen on the surfaces.

The behaviour of a flapped airfoil in transonic flow is also of great interest. There are lessons to be drawn by examining the shock movements as the flap is raised and lowered.

The photo at left shows the flapped airfoil in transonic flow, with the flap un-deflected.



The image at left is a repeat of the conditions above. However, the flap at the trailing edge has been raised slightly. A shock has appeared on the lower surface, with a smaller shock on the flap hinge-line.

Now to summarise the effects seen above in the tunnel photos.

1. At low airspeeds, less than say 150 m/s, compressibility effects are barely detectable.

2. As speeds increase, from roughly 150 m/s to 250 m/s, compression of the air becomes apparent at the stagnation point. Immediately to the sides of the stagnation point, low pressures occur, leading to an expression of the form “leading edge thrust”.

3. At speeds above 250 m/s, shocks may occur, dependent on the shape of the airfoil.

4. At moderate angles-of-attack, the shock appears first on the upper surface.

5. As airspeed increases further, shocks may appear on the lower surface.

6. Shocks appear more readily on thick airfoil sections than on thin airfoil sections.

7. Deflection of a flap at the trailing edge has a strong effect on both the intensity of the shocks and on their location.

The photos showing shocks at Mach 1 and higher are there just for fun. We are only interested in finding the speed at which the shocks first appear: that is, in finding the critical Mach number. The properties of airfoil thickness and airfoil angle-of-attack strongly influence the critical Mach number.

Time to move on again. The next section, Part 8, is on “similitude”, a rather ill-defined topic that is of central interest to us, as we must use incompressible flow data to find the critical Mach number in compressible flow. Tricky.

Critical Mach # Part 8: **Similitude**

There is a word I like very much but didn't really know its meaning. That word is "verisimilitude". The Concise Oxford English Dictionary gives the meaning, "the appearance or semblance of being true or real". That covers a lot of territory in modern life! Now I am in the business of using "incompressible" data to derive "compressible" data, quite a leap of faith. It requires great fortitude, strength of character and duplicity on my part to make this leap, so here we go!

Here is a relevant quotation from Godsey and Young (Ref. 1), which avoids my having to compose one.

"Without theoretical guidance as to the expected variation of performance with flight conditions, it would, for example, be impossible to extrapolate sea-level bench tests or low-altitude flight test results to predict expected high-altitude flight performance. The necessary guiding principles are provided by dimensional theory and **the laws of similarity**."

There are difficulties in placing large aeroplanes in wind tunnels. Not only that, enormous amounts of power are required to blow air over them. For this reason, timid engineers use small models instead of the real thing for testing the forces acting on the full-size airplane. They found that exact scale models, that is, models geometrically the same as the full-size, did not give the correct force parameters. Adjustments had to be made so that the forces measured matched those of the actual airplane. These adjustments involved corrections for Reynolds number and Mach number (Ref. 2).

In other words, "geometric similarity" had to be replaced by "dynamic similarity". The latter term applies to the pressures present on the surfaces of the airplane/model.

A third form of similarity is "kinematic similarity", where the velocities over the surfaces are scalable. Not too surprisingly, there can be overlap between these different forms of similarity, so we will (conveniently) not be too didactic in application of these three concepts.

There exists a way of scaling airfoils for dynamic similarity. The Prandtl-Glauert scaling rule takes an airfoil tested at low velocities, scaling the airfoil for flow in high velocity air. That is, the data from an airfoil tested in "incompressible" flow conditions will apply directly to the scaled airfoil in "compressible" flow. The lift coefficient, lift-slope and all force coefficients (but not drag) retain the same values for the scaled airfoil as for the parent airfoil (Ref.3). This is all a bit too good to be true. If it were not that the measured data, from a wind tunnel (Ref. 4), agreed with the results from the scaled airfoil, then we would have to suspend belief entirely.

Now we must suspend disbelief.

The critical Mach number depends on the local flow speed over the airfoil surfaces. If one could establish "kinematic similarity", in the same way that the Prandtl-Glauert rule established "dynamic similarity", then the use of the incompressible (low speed) data could be justified for establishing data for incompressible (high speed) flow. Rather nice.

So I took a closer look at where the Prandtl-Glauert rule came from. The fluid dynamic tool used is an analysis of the potential flow problem associated with a slender airfoil of low camber. The method provides simplification by throwing away parts of the maths that have small, possibly negligible, values. This is called "linearization". Now here is the fun part.

Potential flow analysis is based on the velocities in the flow field. That is to say, the speeds of the air over the airfoil! In other words, to obtain "dynamic similarity", Prandtl and Glauert first established "kinematic similarity"! The job was already done for me!

Now for an irrelevant interlude. Earlier I expressed a view that "similarity" might not be too well defined, at least, not precisely. You be the judge. Below is a quote from an author who shall remain anonymous.

"If points that correspond to each other on model and prototype are called corresponding points, and particles in motion that correspond to each other are called corresponding particles, then the motions in the two systems are similar if corresponding particles move at corresponding points at corresponding times".

How could you argue with that? In case you are still wondering, this was a definition of kinematic similarity. Not that I could do any better.

The time has come to have a closer look at the Prandtl-Glauert rule, in Part 9.

1. F. W. Godsey and Lloyd A. Young (1949). "Gas turbines for Aircraft", p23. McGraw Hill.
2. E. L. Houghton and A. E. Brock (1960). "Aerodynamics for Engineering Students", p68. Edward Arnold.
3. L. M. Milne-Thomson (1958). "Theoretical Aerodynamics", p276-278. MacMillan.
4. Ira H. Abbot and Albert E. von Doenhoff (1959). "Theory of Wing Sections", p257. Dover Publications Inc. SBN 486-60586-8.

Critical Mach # Part 9: **Prandtl-Glauert rule.**

Previous authors have described the Prandtl-Glauert rule most eloquently. First, Abbott and von Doenhoff (Ref. 1).

"The Glauert-Prandtl rule relates the lift coefficient or slope of the lift curve of a wing section in compressible with that for incompressible flow. This relation was derived for the case of small disturbance velocities and low Mach numbers. There conditions are approximated for thin wing sections with small amounts of camber at low lift coefficients at speeds well below the speed of sound. The Glauert-Prandtl relation is

$$cl_c / cl_i = a_c/a_i = 1/\sqrt{(1-M^2)}$$

where the subscripts c and i denote, respectively, the compressible and incompressible cases.

The Glauert-Prandtl rule agrees remarkably well with experimental data, considering the assumptions made in its derivation".

Rather more usefully, Katz and Plotkin (Ref. 2) write:

"The first and most straightforward modification to an incompressible potential-flow based method is to incorporate the effects of "low-speed compressibility" (e.g., for $M_\infty < 0.6$). This modification can be obtained by using the Prandtl-Glauert rule, as developed in Section 4.8. Thus, small disturbance flow is assumed, and a compressibility factor β can be defined as

$$\beta=\sqrt{(1-M_\infty^2)}$$

If the free stream is parallel to the x coordinate then the x coordinate is being stretched with increased Mach number while the y and z coordinates remain unchanged."

A nice derivation of the Prandtl-Glauert rule is to be found in Miles book, "Supersonic Aerodynamics". A point made by Miles (Ref. 3) is that the derivation uses the free stream Mach number M_∞, rather than the local Mach number.

In applying the Prandtl-Glauert rule to propeller airfoil design, one follows the following steps:

1. Choose a favourite airfoil for which one has low speed data.

2. Determine the free stream Mach number at the selected blade element.

3. Set the airfoil at the angle-of-attack for the desired lift coefficient.

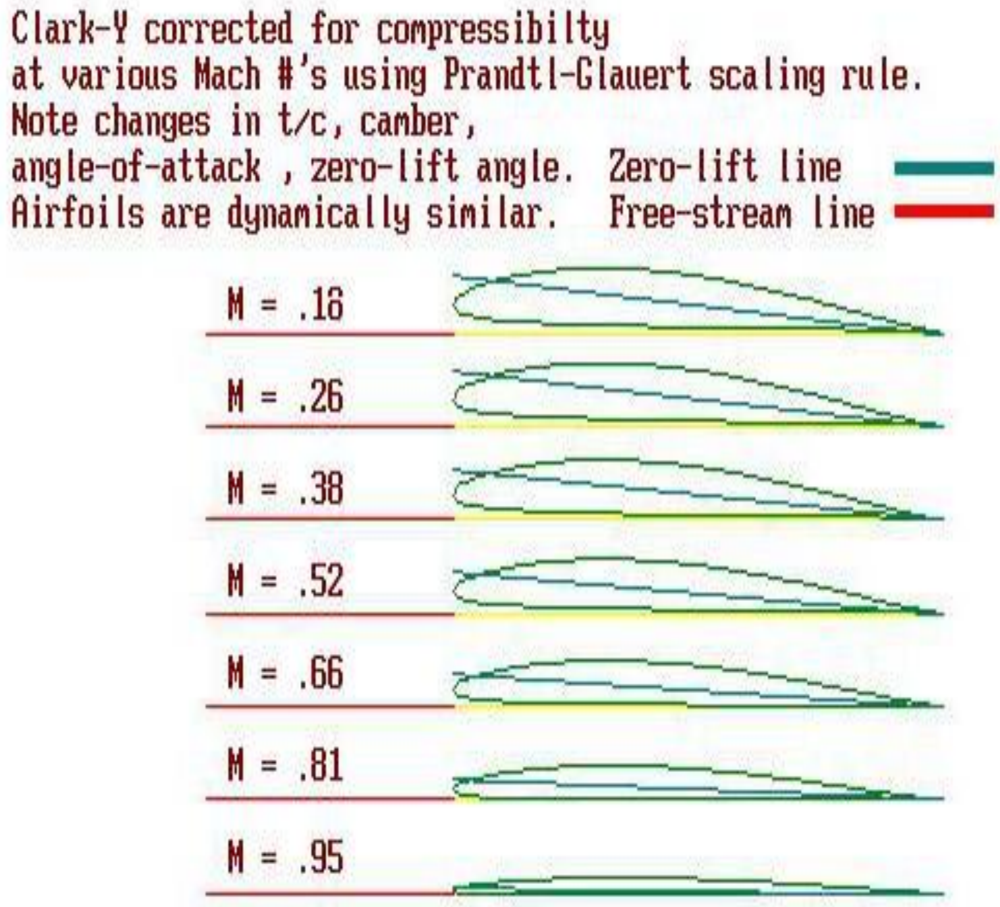
4. Scale (stretch) the x coordinate according to β^{-1} .

5. Re-scale x and y coordinates linearly to give the desired chord.

6. Compute the angle-of-attack to chord-line of the stretched airfoil.

7. Proceed to use the previously selected lift coefficient.

The figure below is intended to show how Clark-Y must be rescaled as a function of M_∞.



At this point, we now have an airfoil for use in compressible flow, complete with its force coefficients (not drag).

The reader may have noticed that the previous authors have limited application of the Prandtl-Glauert rule to free stream flow speeds below approximately Mach .7. There is nothing in the derivation of the rule to suggest this limitation, other than when $\beta = \infty$ which occurs when $M_\infty = 1$. I often wonder if this condition prompted the invention of the "Sound Barrier".

The real reason is that the rule fails to work because shock waves may form above Mach 0.6, leading to severe loss of lift and high drag.

Our challenge, then, is to raise the velocity at which shock waves form as high as possible. Since there are known airliners, such as the Convair 990, which can fly at Mach .9, or even World War 2 fighters that could dive vertically at Mach 0.8 (well, Spitfire, anyway), our prospects of success are good. Good, that is, if we can understand why the shock waves form in the first instance.

Readers with a grasp of Fluid Dynamics are prompted to refer to Milne-Thomson's (Ref 4) very useful chapter "Subsonic and Supersonic Flow" in the references below. Not only does he derive the Glauert correction factor, he also quantifies the density change in compressible flow, then provides a method for calculating the streamlines in compressible flow based on the streamlines in incompressible flow.

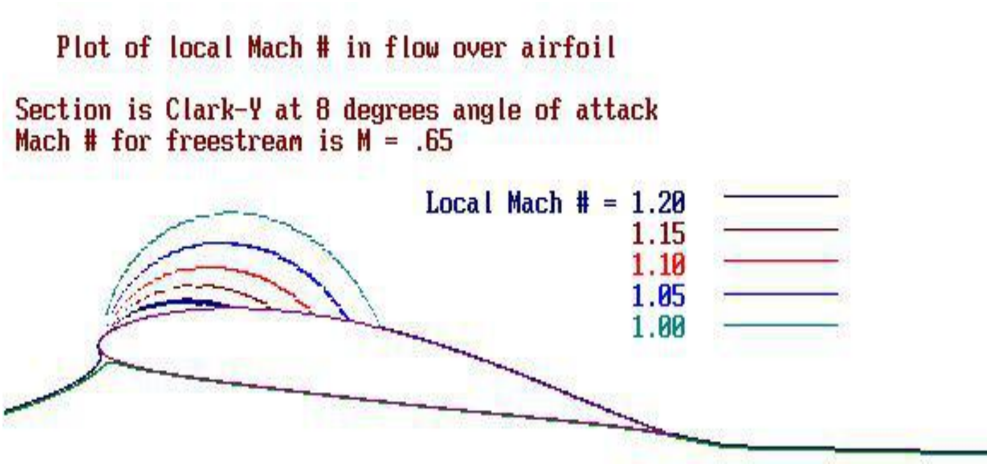
Turn to Part 10.

References:

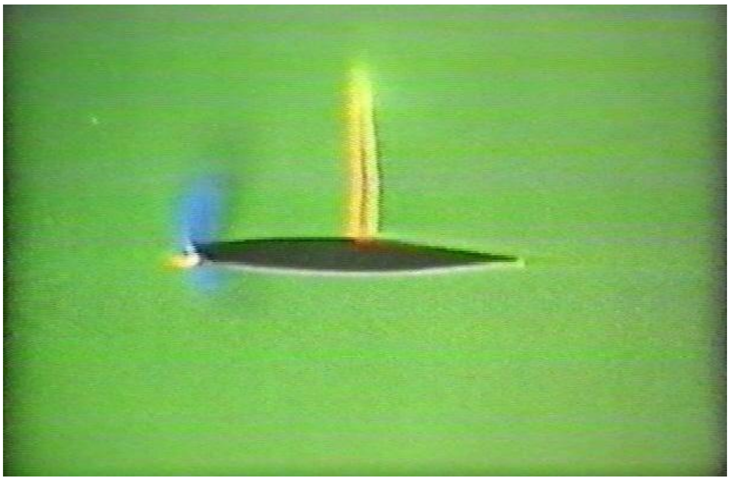
1. Ira H. Abbot and Albert E. von Doenhoff (1959). "Theory of Wing Sections",p256,257. Dover Publications Inc. SBN 486-60586-8
2. Joseph Katz and Allen Plotkin (2001). "Low-Speed Aerodynamics", p227. Cambridge University Press.
3. Edward R. C. Miles (1950). "Supersonic Aerodynamics", p104-107. McGraw-Hill.
4. L. M. Milne-Thomson (1958). "Theoretical Aerodynamics", p267-278. MacMillan.

Critical Mach # Part 10: **The mechanism for shock wave formation**

The time has come to bite the bullet. Exactly what is a shock wave: how is it formed? The figure below shows flow over an airfoil at several free-stream Mach numbers, all less then unity. The subsonic free-stream flow is seen to accelerate over the upper surface, become supersonic for a certain distance; then slowing down again to subsonic speed. Unfortunately, this simple picture is wrong.



The figure, below, better illustrates the real-world cussedness of transonic flow. As speed is increased, a small region is reached where the speed of flow is supersonic. Instead of then slowing down nicely, there is a sudden change in the behaviour of the air. A shock wave grows out from this point, growing taller with increased intensity as the speed continues to increase. This is the weird behaviour we need to understand.



Here is what Godsey and Young wrote, back in 1949 (Ref. 1).

'For free-stream velocities so low that the velocity of sound is not locally reached anywhere on the airfoil, disturbances originating at or near the trailing edge travel forward and affect the entire flow pattern. With increasing velocity, the local flow Mach number takes on the value unity at some point on the airfoil. A shock wave then starts to grow from this point. Downstream disturbances now are unable to travel through this shock, so that the motion ahead of the front is independent of what happens behind it. The shock has a profound affect, however, on the flow over the after portion of the airfoil. The sharp pressure change at the wave front affects the motion in the boundary layer, thickening it and promoting separation'

Now for Ray Whitfords version, made in 1987, 38 years later (Ref. 2).

'The varying accelerations applied to the air particles passing over and under the wing result in velocities higher than the flight velocity on the upper surface and lower (than the flight) velocity on the under surface. As the speed of the aircraft approaches the speed of sound, the higher velocities on the wing's upper surface will reach and eventually exceed the local acoustic velocity, producing regions of supersonic flow. The usual way for such a flow to decelerate back to the flight speed is via a shock wave. This is a very thin pressure wave pulled through the air by the wing and across which extremely abrupt changes of flow velocity occur. Static pressure, temperature and density of the air all increase through a shock wave, while the (local) velocity (of the air) and Mach number drop. There is also a drop in total pressure. This is the term which is equated to the static and dynamic pressure in the Bernoulli equation. As a result, the equation cannot be applied across the shock wave, since one of the assumptions is that the flow is continuous (i.e. flow properties change gradually). Across a shock wave the flow is anything but continuous. The loss of total pressure adds up to a loss of momentum which gives rise to wave drag.'

Well, all that was quite a mouthful. The authors quoted above have certainly given a good description of the shock wave phenomena, but are we any the wiser as to the mechanism producing the shock wave? Not I. (but then wisdom is certainly lacking in my mental makeup anyway!). So are there any clues in the above descriptions which might help us?

Both accounts seem to be worried by the speed of sound. Evidently that is the root of all evil. Whitford mentions the little particles of air which bang their way over the wing surface, and rubbishes the Bernoulli equation. Not much to go on, is there! It is all downhill once the shock wave forms. We are looking for a mechanism for shock wave formation. Recall in Part 2 of this discourse, I felt inclined to recall Newtons laws of motion. Now those laws provide pretty good mechanisms for nearly everything. Perhaps if we apply these laws to Whitford's particles and muddle in the speed of sound we could be on the right track. First we need to get a better handle on these little particles.

Firstly, there are a lot of them. When I was a boy, there were 6.023×10^{23} molecules in a mole of air. Forget what a mole is, just look at the number of molecules: it is stupendous. Recently, I read a chemistry text which said there were 6.0×10^{23} molecules in a mole. What happened to the other 0.023×10^{23} molecules? This huge number is the reason we can treat air as a fluid, rather than just lots of little pieces. If we want to understand the behaviour of the fluid, we are going to have to return to the behaviour of these little chaps.

Next, there are big gaps between the molecules of air. If we wanted to squish all the molecules together so that were no gaps between them, then we would have to reduce their volume by something approaching a thousand fold. (more accurately, 700 fold). Nothing like that compression of the volume occurs over an airfoil. A factor of two might be possible, but not seven hundred. So possibly compression alone will not be of much use to us.

Finally, the little molecules are bouncing about, quite often colliding with each other. Sometimes in a collision, one of the molecules might stop altogether, just as with billiard balls. The other partner in the collision then dashes off with extra speed. Whatever, there must be some average speed for these molecules. Would you believe that speed is about 500 m/s: that is faster than the speed of sound, which is roughly 340 m/s! The distance travelled on average between collisions is less than one millionth of a metre, there being roughly 10^{10} collisions per second in a mole of air (Ref. 3).

Now we could lose sight of just what "flow" of molecules, all bouncing about, actually is. The general streaming motion of air is the statistical mean of all the different velocities of the various particles. The thermodynamic measure of all this bouncing about is the temperature. We have already seen in Part 3 (on thermodynamics) that temperature, pressure, density, flow velocity and the speed of sound are all connected to each other; that is just the nature of air. Now we find that all these macroscopic quantities are intimately connected to the way the molecules are bouncing about. Not only that, they are bouncing about with speeds that are on average faster than the flow itself. Perhaps we have here a mechanism for explaining the Bernoulli equation.

Recall (from Part 2) that the Bernoulli equation says that fast moving air over an airfoil has reduced pressure. This is not an "obvious" situation. But if the effect of passing air over an airfoil is to redirect the random bouncing about of the molecules in the direction of a stream line, then we can see that the molecules can use their excess thermal speed to go fast along the streamline. Hence speeding up over the airfoil is a thermal effect!

Not only that, as the molecules are now more often directed along the streamline, rather than just all over the place, fewer of the molecules will be striking the airfoil surface. Thus increased speed and reduced pressure go hand in hand.

Once the flow has passed the upper surface highpoint, the effect of the airfoil is no longer to accelerate the flow. The thermal motion re-asserts itself, the velocity reduces and the pressure rises. But now we must ask: what happens when the flow is so fast that most of the random thermal motion has been changed into directional motion? How is the flow to expand back into high pressure aft of the high point?

Well, it can't. We have hit the wall, at the point where the flow velocity is the same as the local speed of sound. The implicit assumption in the Bernoulli equation is that the thermal motion of the molecules will permit the exchange of pressure into speed, and speed into pressure. With flow at or above the speed of sound, this exchange can no longer occur.

Rather, there is the dramatic change in flow listed above by Whitford. Within a very small distance, comparable to the mean path between molecular collisions, everything changes. This is not a smooth process: it is a discontinuity, a dramatic one at that. There is an almost instantaneous change of momentum in the flow. Recalling Newtons laws, a change of momentum defines a force: in this case a retarding force which must be overcome with power from the engine.

The temperature rise behind the shock wave cannot be easily connected by the gas equation. There is an excess of temperature rise: the associated thermal energy cannot be recovered. The extra useless, random motion has a name: entropy.

Hopefully this rather long-winded discussion has shed a little light on the reason shock waves form. More to the point, we can see why we do not want shock waves: we must take steps to reduce or eliminate them. Not easily done; however, if we can delay their formation that might be enough. We are back to wanting to raise the critical Mach number of the flow over an airfoil.

The next section, Part 11, goes through a process for modifying the shape of an airfoil so that shock wave formation is delayed to higher free-stream Mach numbers.

References:

1. F. W. Godsey and Lloyd A. Young (1949). "Gas turbines for Aircraft", p26. McGraw Hill.
2. Ray Whitford (1987). "Design for Air Combat", p17, Fig.3. Janes. ISBN 0 7106 0426
3. Charles Kittel (1969). "Thermal Physics", p206-210. John Wiley and Sons. SBN 471030 X

Critical Mach # Part 11: **Airfoil Selection for propellers**

Just in case you have forgotten, a reminder. We are trying to minimise the losses associated with shock wave formation near propeller tips. The method is to try to find airfoils which delay to a higher Mach number the start of shock wave formation. The main tool we have is the Oshkosh Airfoil Program (Ref. 1) which gives us a pretty comprehensive description of airfoil properties in incompressible flow. Application of the similarity principle, by invoking the Prandtl-Glauert scaling rule, provides a fair idea of the airfoil properties in compressible flow, which is what we have when air flows over an airfoil at high speed.

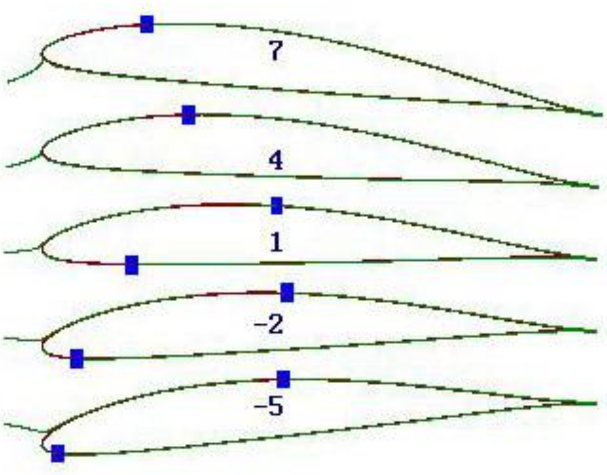
These processes I have coded into a Quick Basic code suite which generates pictures; hopefully the pictures will help make matters clear. First a definition. A "parent" airfoil is the starting point for the method. In the examples below, the famous Clark-Y airfoil was chosen as a "parent", with the airfoil being analysed in incompressible flow. The "parent" airfoil is NOT used on the actual propeller. After transformation by the Prandtl-Glauert rule, the Clark-Y becomes a "daughter" airfoil. The "daughter" is then used for manufacture of the propeller.

The "daughter" airfoil will differ from the "parent" airfoil, in camber, thickness and the operational angle-of-attack. The only parameter shared without change by the "parent" and "daughter" airfoils is the lift coefficient.

In the diagram below, which shows the "parent" airfoil at different angles-of-attack, the velocity distribution has been found from the streamline running along the airfoil surface. The blue boxes show where shock waves initiate as the angle-of-attack to the free-stream is increased. The shock wave starts to form when the local velocity of the air exceeds the local speed of sound: the velocity of the air at this point is called the critical Mach number.

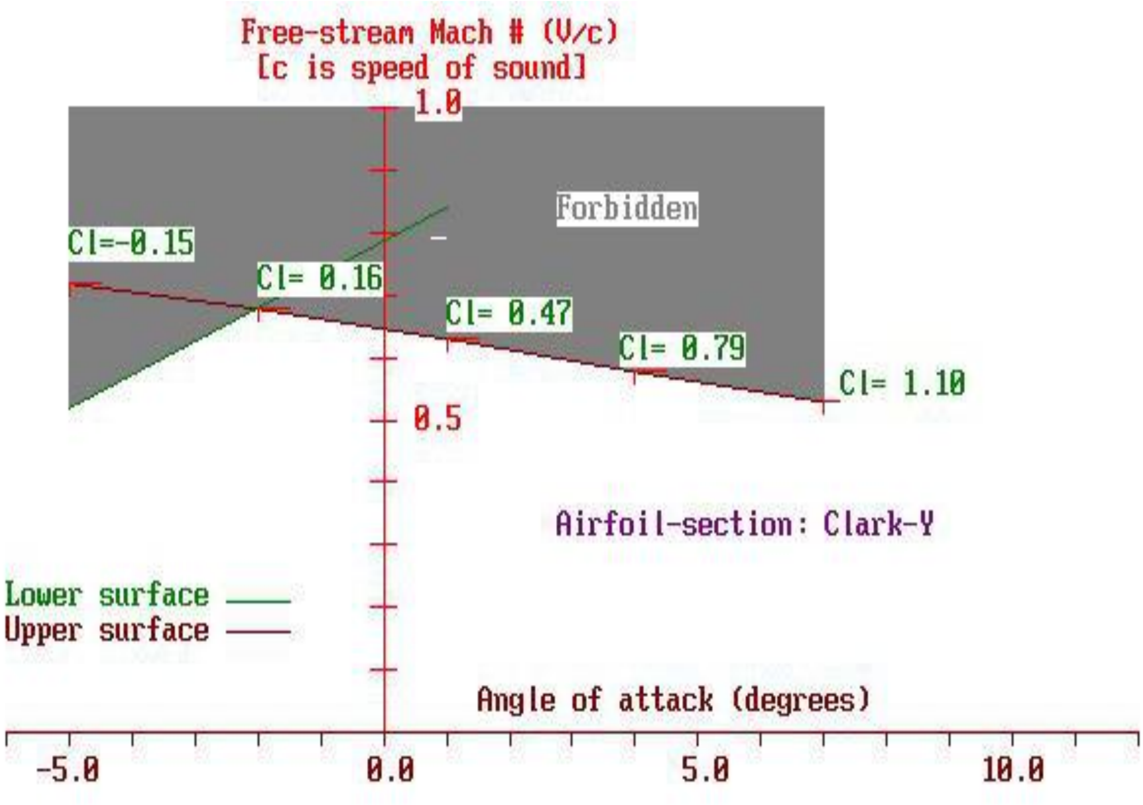
Worth noting is, at a sufficiently low angle-of-attack, that shock waves can initiate on the lower surface: there can even be shock waves present on both upper and lower surfaces at the same time.

We are not interested in further growth in intensity of the shock waves: our intention is simply to delay the initiation of the shock.



Clark-Y airfoils showing the location of the critical Mach number for various angles-of-attack. Clark-Y is the "parent" airfoil. The calculations are performed as though the air were incompressible.

The Oshkosh code not only delivered the streamline, but also the lift coefficient and zero-lift angle for the airfoils above. The data for incompressible flow are displayed below.



There is a lot on this chart, so look closely. Starting with the axes, we have the abscissa as the angle-of-attack in incompressible flow. The ordinate is the free-stream Mach number. The points for our set of Clark-Y airfoils above are marked in blue, with their corresponding lift coefficients written in next to them.

The gray area to the left of the abscissa are mostly the result of shock waves forming on the airfoil lower surface, while to the right of the origin the gray areas are related to shock wave formation on the airfoil upper surface.

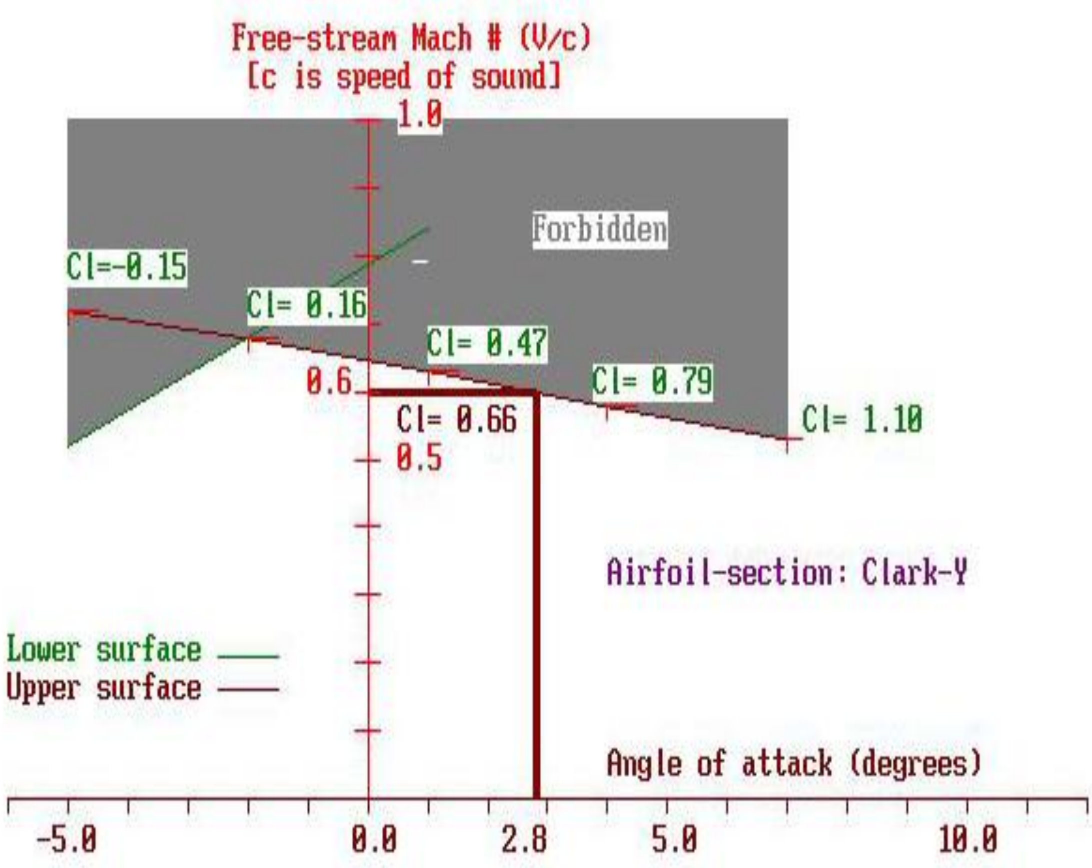
All of the gray area is "forbidden". If you chose a Mach number or angle-of-attack which places your airfoil in this area, then there will be shock waves formed on that airfoil. Performance degradation will commence, along with an increase in radiated noise levels. Our intention is to stay out of the gray region in our propeller selection.

Recall again that the above chart is based on incompressible flow data. This chart applies to the "parent" airfoil. However, the ordinate, which is the Mach number, clearly has values where the compressibility of air cannot be ignored. So we have a paradox.

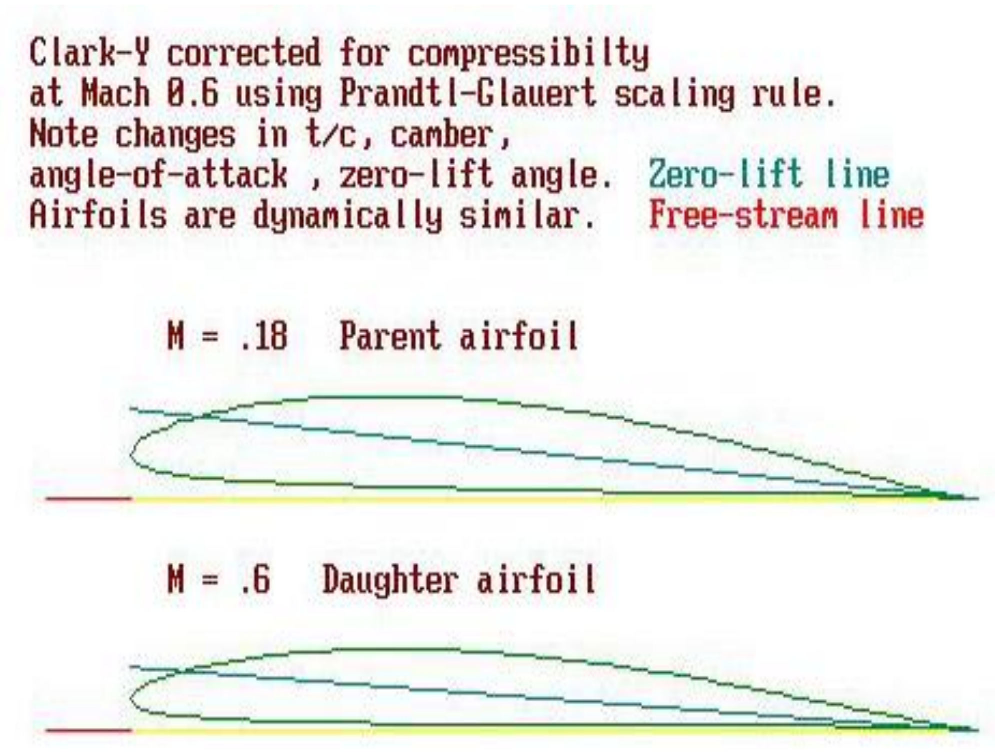
Now we invoke the similarity principle. If two airfoils have the same pressure coefficients, e.g. lift coefficients, then they are both kinematically and dynamically similar. They are thereby scalable by the Prandtl-Glauert rule. Alternatively, we can say that the geometry of the "daughter" airfoil can be found by a transformation performed on the "parent" airfoil.

We now re-draw the figure above with an operating point marked in place.

The lines marked in red are the chosen operating point. That is, we choose a point out along the radius of the propeller where the airflow speed is Mach 0.6. For a propeller with a tip speed of Mach 1, this will be 60% of the radius out from the axis of rotation.



Below are drawn the “parent” and “daughter” airfoils for the case above.



The "parent" airfoil is Clark-Y at Mach .18, due to forward motion of the aircraft.

Now we need to find the angle-of-attack and zero-lift angle for the "daughter" airfoil. Might as well, for this is the airfoil for our propeller blade element operating at Mach 0.6. To repeat, the "daughter" airfoil has been scaled from the "parent" airfoil using the Prandtl-Glauert rule.

Reading from the diagram for the operating conditions above, we have at Mach 0.6 that the lift coefficient is 0.66 which is attained at an angle of attack α of 2.8 degrees. Being right on the edge of the gray area, no shock wave has formed. The section chosen is again Clark-Y: the flow is again incompressible. Yet at Mach 0.6, the flow truly is compressible!

If we chose to accept the lift coefficient of 0.6 as being satisfactory, then we must now find the true airfoil, the "daughter" airfoil. By applying the Prandtl-Glauert scaling rule to the Clark-Y section, we arrive at the plots above. The scaling process also produces angle-of-attack and zero-lift angles which apply for the compressible flow at Mach 0.6. Proceed as follows. Recall that the Prandtl-Glauert scaling factor β is given by:

$$\beta = \sqrt{(1-M_\infty^2)}$$

With M_∞=.6, $\beta = \sqrt{(1-.6^2)} = .8$

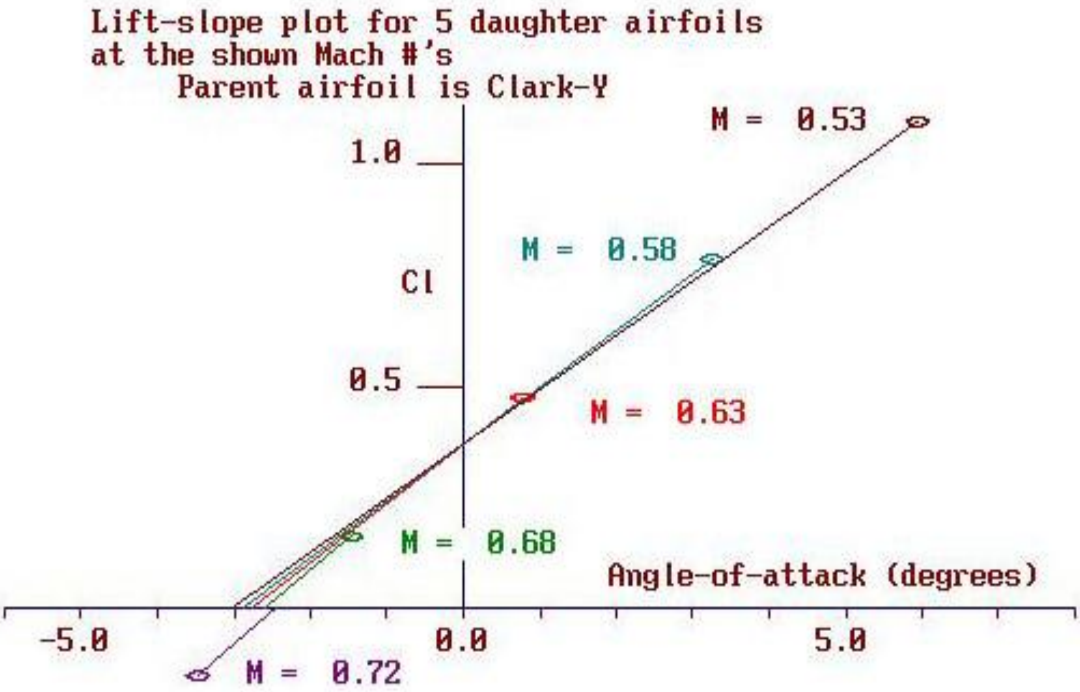
the angle-of-attack of the "daughter" airfoil α_d (at chordline) is to be set to:

$$\alpha_d = TAN^{-1}(TAN(\alpha_d) / \beta^{-1})$$

$$= TAN^{-1}(TAN(2.8*\pi /180) / .8) *180 / \pi$$

$$= 2.24^\circ$$

From the Oshkosh program, the zero-lift angle α_{d0} for Clark-Y was -3.54° so by the same math, α_{d0} = -2.83°. This is interesting; we now have 2 points on the lift-slope line, the upper point of which is cut off by the shock wave forbidden zone. Since the lift-slope line is straight, we can connect these points to display the allowed intermediate lift coefficients as a function of angle-of-attack in compressible flow.



There are 5 different Mach numbers, requiring 5 different "daughter" airfoils. At the high-end of each line is a circle. Use of lift coefficients above this circle will produce shock waves. Hence angles-of-attack which produce higher lift coefficients than those above are forbidden! (The example we analysed for M = 0.6 is not on the above plots).

The choice of Clark-Y, including its "daughters", is going to lead to shock waves, with their associated poor performance, when the free-stream Mach number exceeds roughly Mach 0.65. This is rather worrying, as the props we use in F2A and F3D get up to tip speeds very close to Mach one. Remember, the "daughter" airfoils have already been thinned due to the Prandtl-Glauert correction for compressibility. Nothing is going to save the Clark-Y "parent".

The need has now arisen to design a new "parent" airfoil for each Mach number on each blade element. Clark-Y is OK up to Mach 0.65: that is the limit of value for Clark-Y.

Looking back into the early 1940's, designers were aware that a better choice of high-speed airfoil was thinner, with less camber. The Miles M52 designer (Ref. 2), Dennis Bancroft, chose to use a 4% biconvex section, for an expected speed of Mach 1.5. This was quite alarming to his contemporaries: nobody knew if an aircraft could actually perform with such an airfoil. Such an airfoil has zero camber. The transonic performance of this airfoil was unknown.

With the tool of the Oshkosh airfoil program at our fingertips, we can easily convert Clark-Y to zero camber, or even negative camber. If it seems crazy to use negative camber, then it probably is. The underlying idea of changing Clark-Y in this fashion is that negative camber reduces the curvature of the upper surface, while also increasing the curvature of the lower surface. Charles Hampson Grant (Ref.3) chose to call these curvatures the "upper camber" and "lower camber". Who could be so churlish as to disagree with the terminology of this great pioneer of misplaced knowledge? My kind of guy!

The net effect of this idea is that we can raise the critical Mach number by lowering the flow velocity over the upper surface, thereby reducing suction: then lower the velocity over the lower surface, thereby increasing pressure on the lower surface. The problem is that this process lowers the lift coefficient to a negative value, requiring a speed up of the free-stream: that joyous occasion then producing a shock near the leading edge on the upper surface!

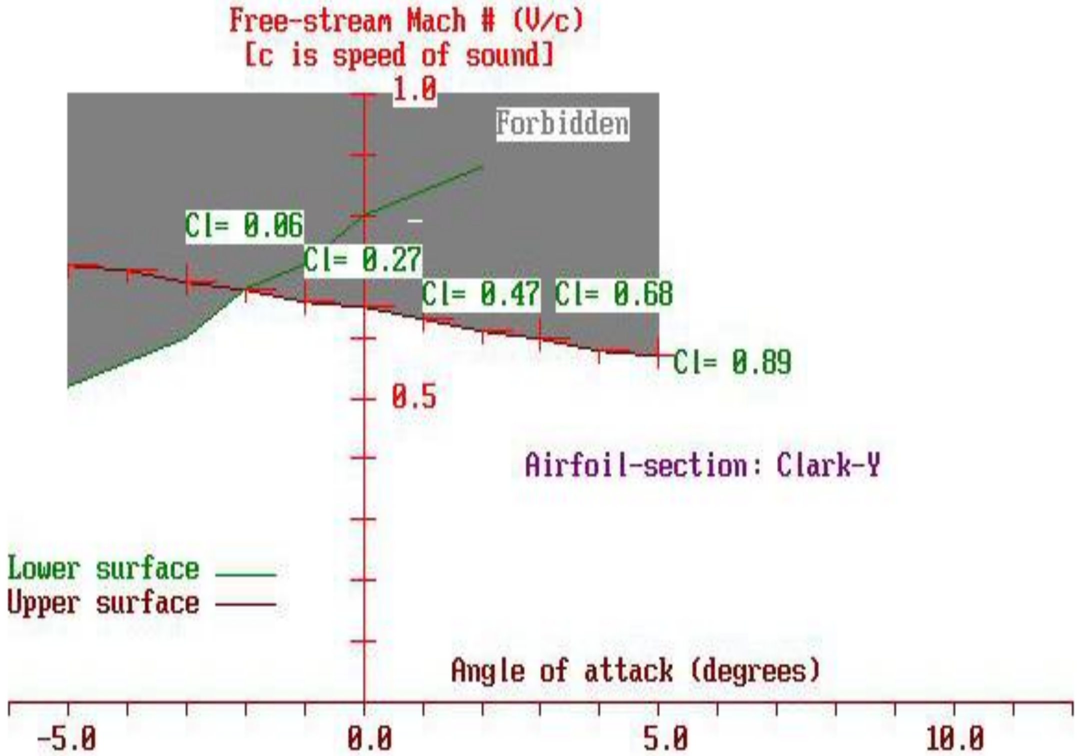
Well, having kicked myself in the guts with that idea, I re-opened R.T. Jones book to find parameters for some intriguing airfoils, one of which was called the "Minimum velocity airfoil". Now that sounds promising for getting at least some rise in the critical Mach number.

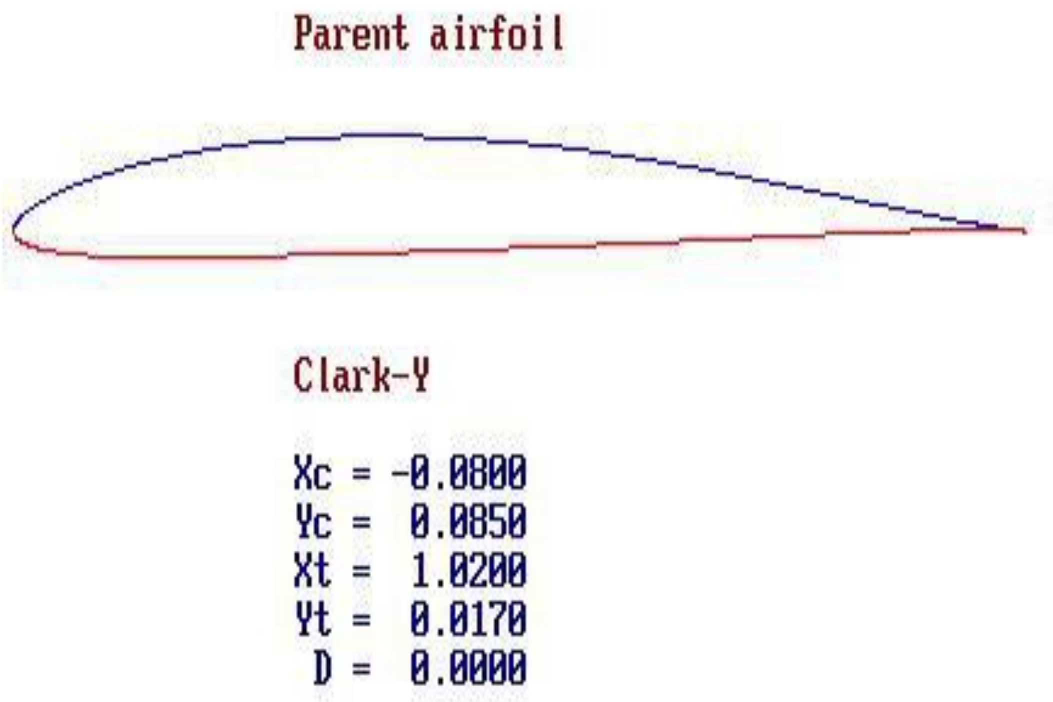
The following pages contain the analysis for the following airfoils:

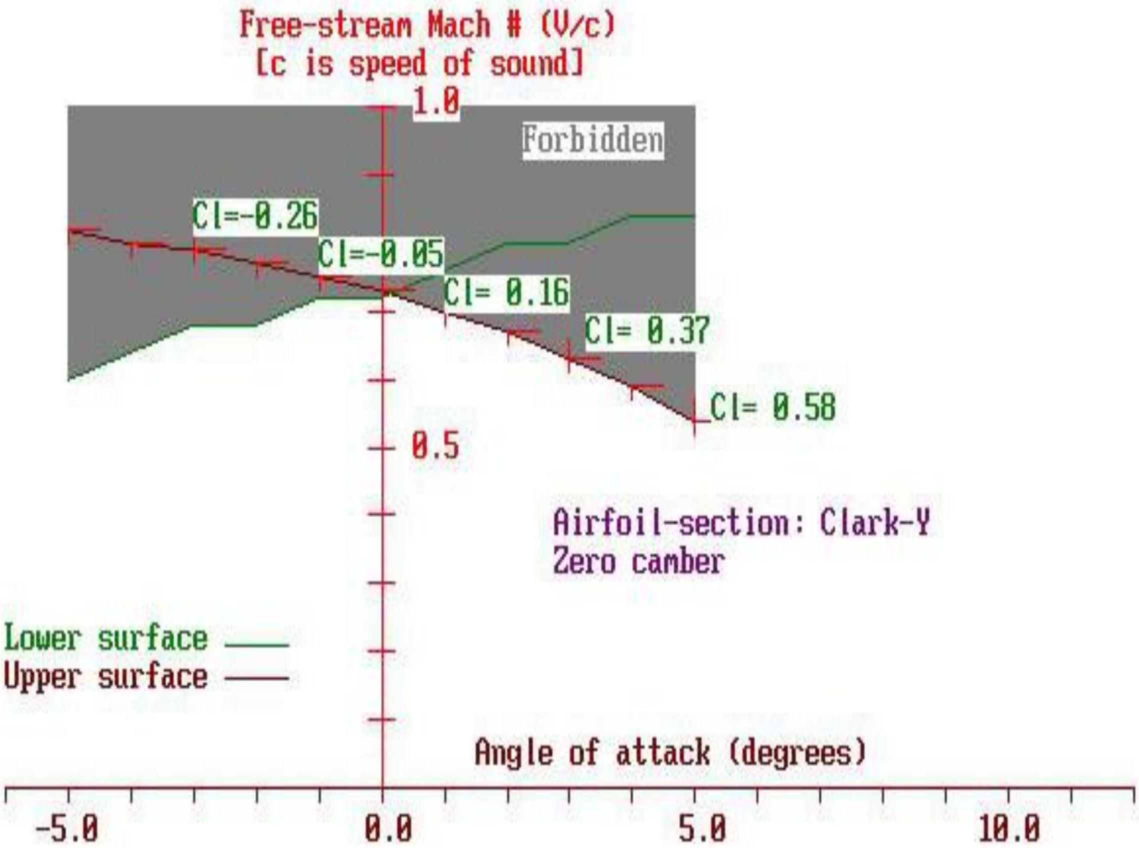
1. Clark-Y
2. Clark-Y with zero camber
3. Clark-Y with negative camber (upside down)
4. A laminar flow airfoil (Jones, Figure 12)
5. A reflex airfoil (Jones, Figure 13)
6. The minimum velocity airfoil (Jones, Figure 15)
7. The minimum velocity airfoil with reduced camber
8. The minimum velocity airfoil with increased camber
9. The minimum velocity airfoil with reduced thickness
10. The minimum velocity airfoil with increased thickness

But why bother? Has Supercool lost his wrinkled lip and sneer of cold command (Ref. 4)? Will the name of Supercool be lost in the withering lone and level sands of time, along with Ozymandias and his ilk? Like a Roo caught in the headlights of a speeding car, will he become just another statistic of faunal road-kill? Have mixed metaphors finally laid this notorious rake in his grave? Or is there something you, dear reader, have missed? Something like scientific rigour?

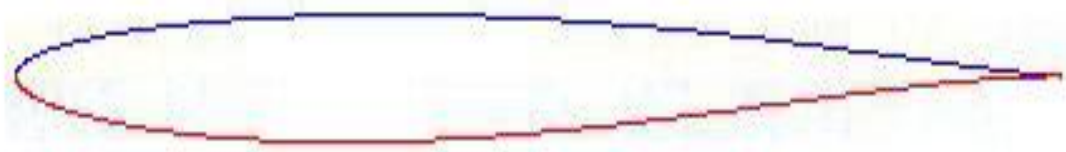
Maybe, but your primitive mind, dear reader, is not yet ready for the final, Agatha Christie-like, denouement. So look closely at the airfoil charts, you may yet solve the riddle.


Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl= 0.06
Cl= 0.27
Cl= 0.47
Cl= 0.68
Cl= 0.89
0.5
Airfoil-section: Clark-Y
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0


Parent airfoil
Clark-Y
Xc = -0.0800
Yc = 0.0850
Xt = 1.0200
Yt = 0.0170
D = 0.0000

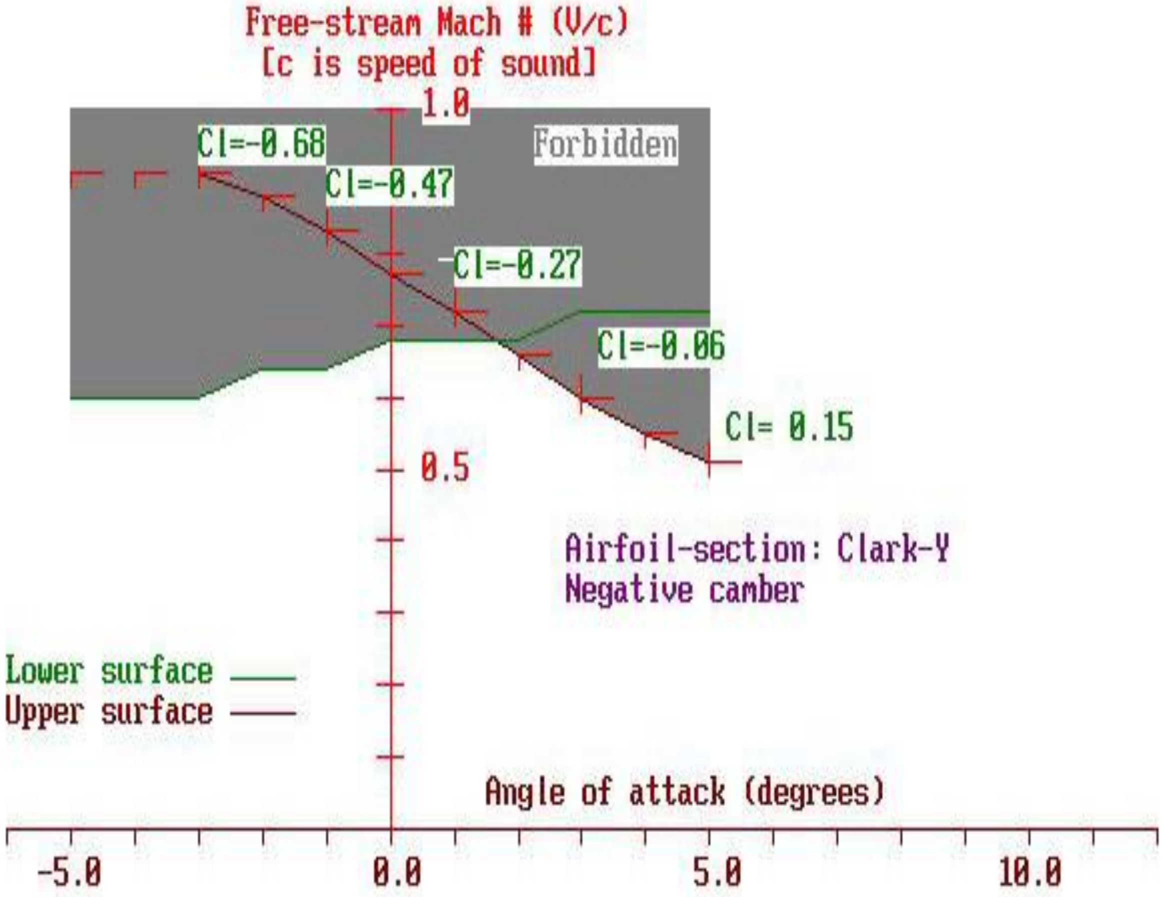


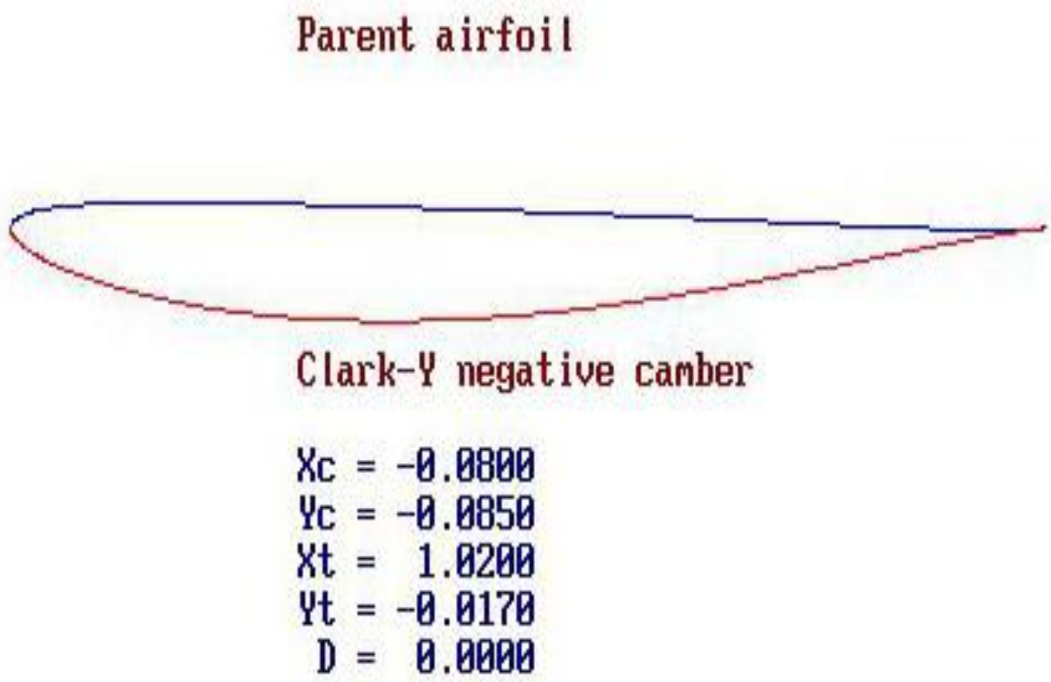
Parent airfoil

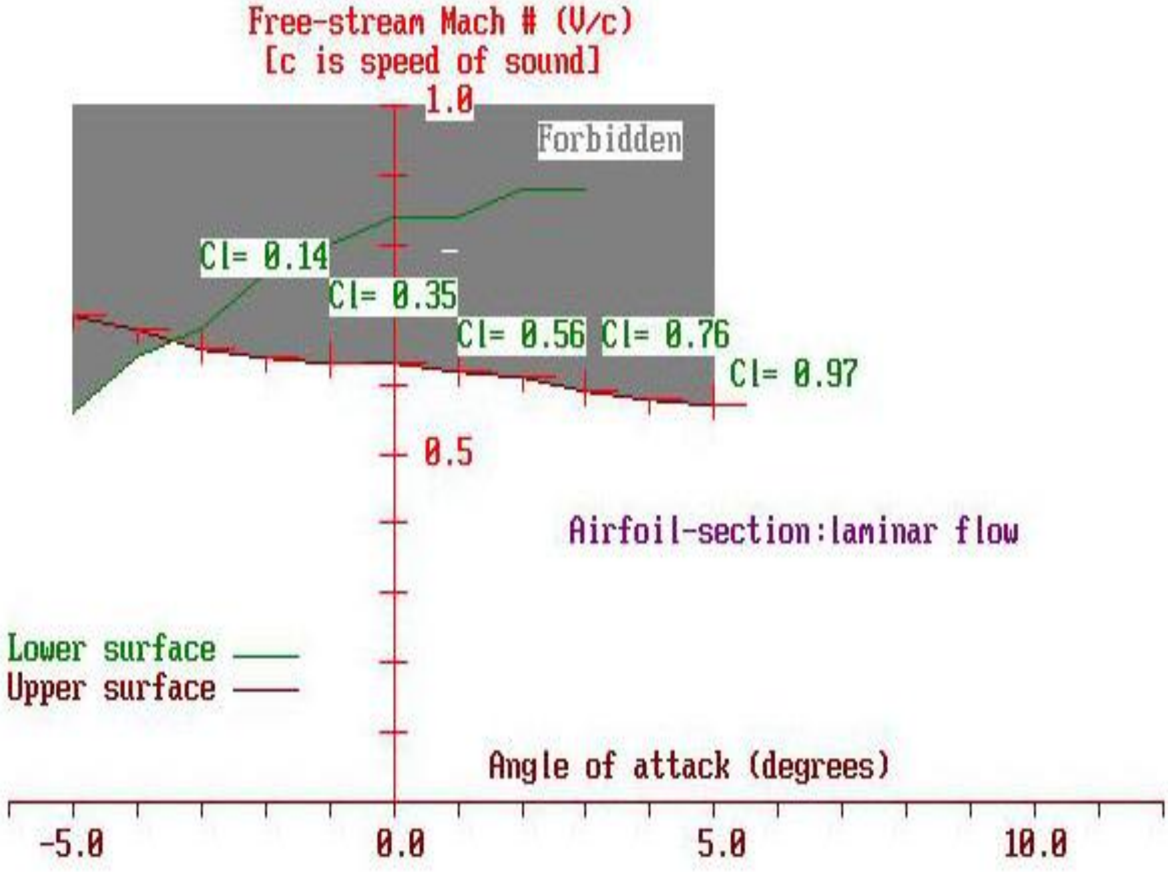


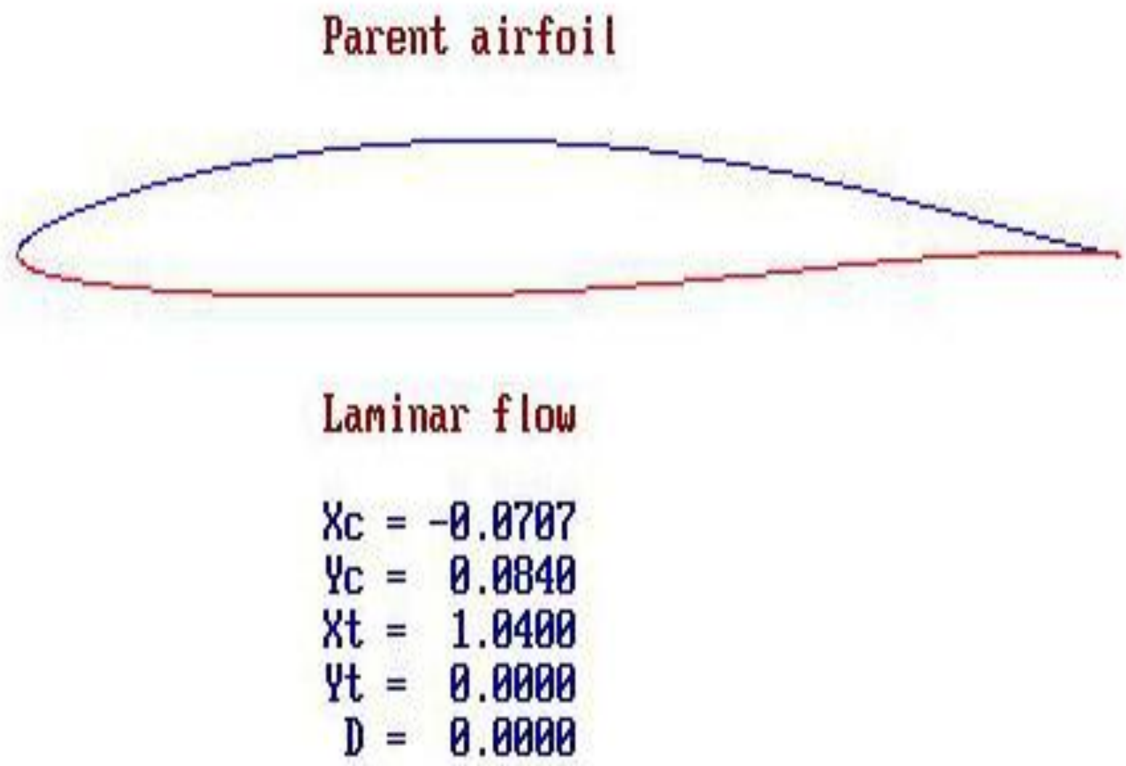
Clark-Y zero camber

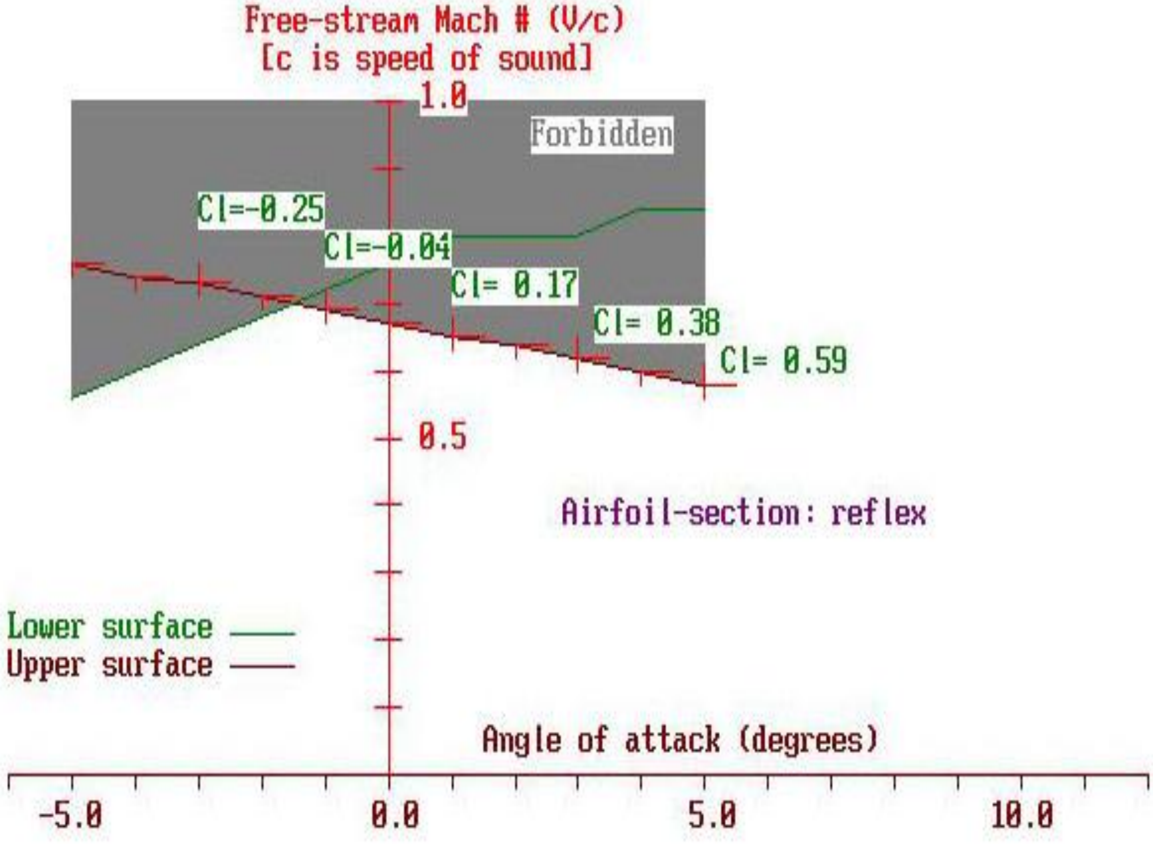
Xc = -0.0800
Yc = 0.0000
Xt = 1.0200
Yt = -0.0100
D = 0.0000

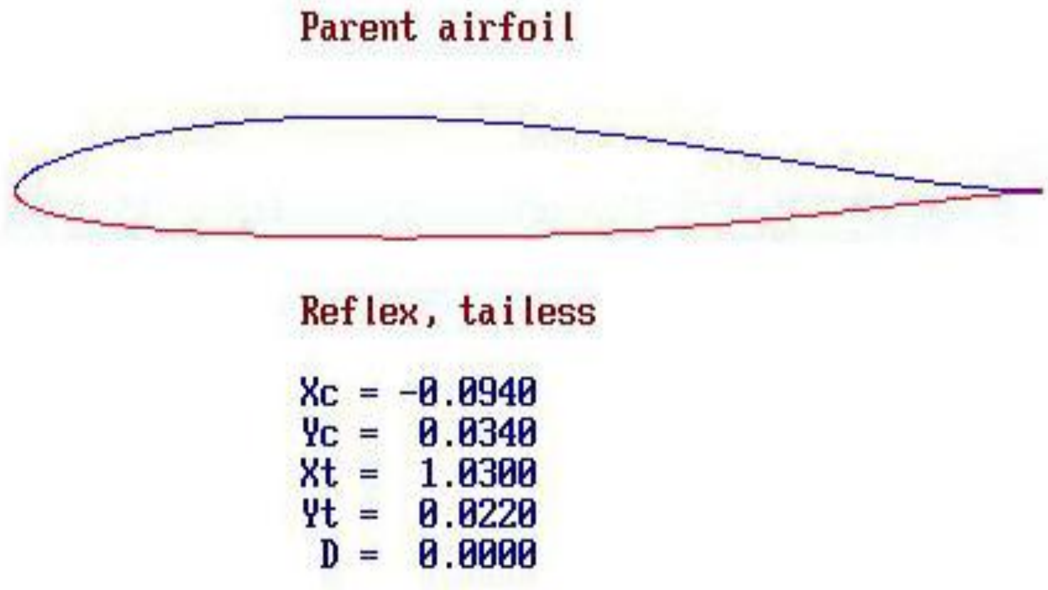

Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl=-0.68
Cl=-0.47
Cl=-0.27
Cl=-0.06
Cl= 0.15
0.5
Airfoil-section: Clark-Y
Negative camber
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0

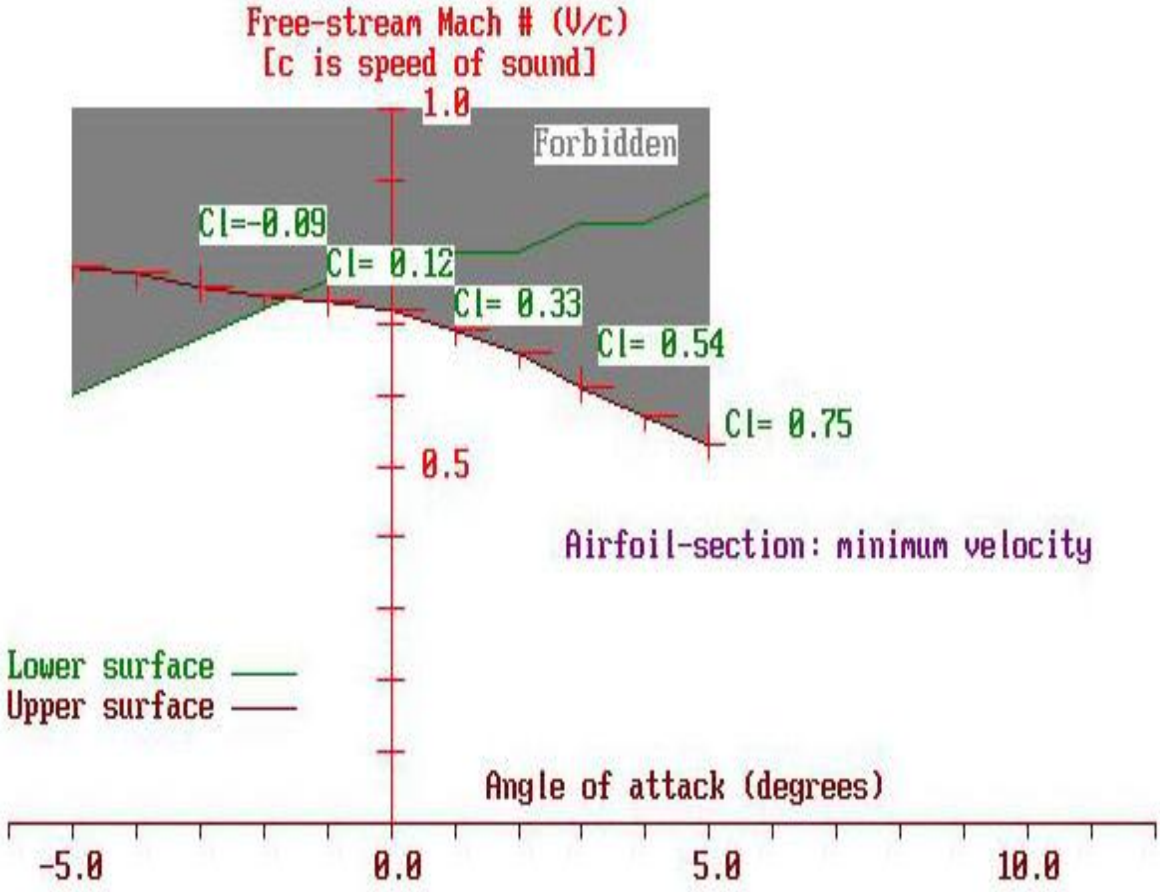

Parent airfoil
Clark-Y negative camber
Xc = -0.0800
Yc = -0.0850
Xt = 1.0200
Yt = -0.0170
D = 0.0000


Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl= 0.14
Cl= 0.35
Cl= 0.56
Cl= 0.76
Cl= 0.97
0.5
Airfoil-section:laminar flow
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0

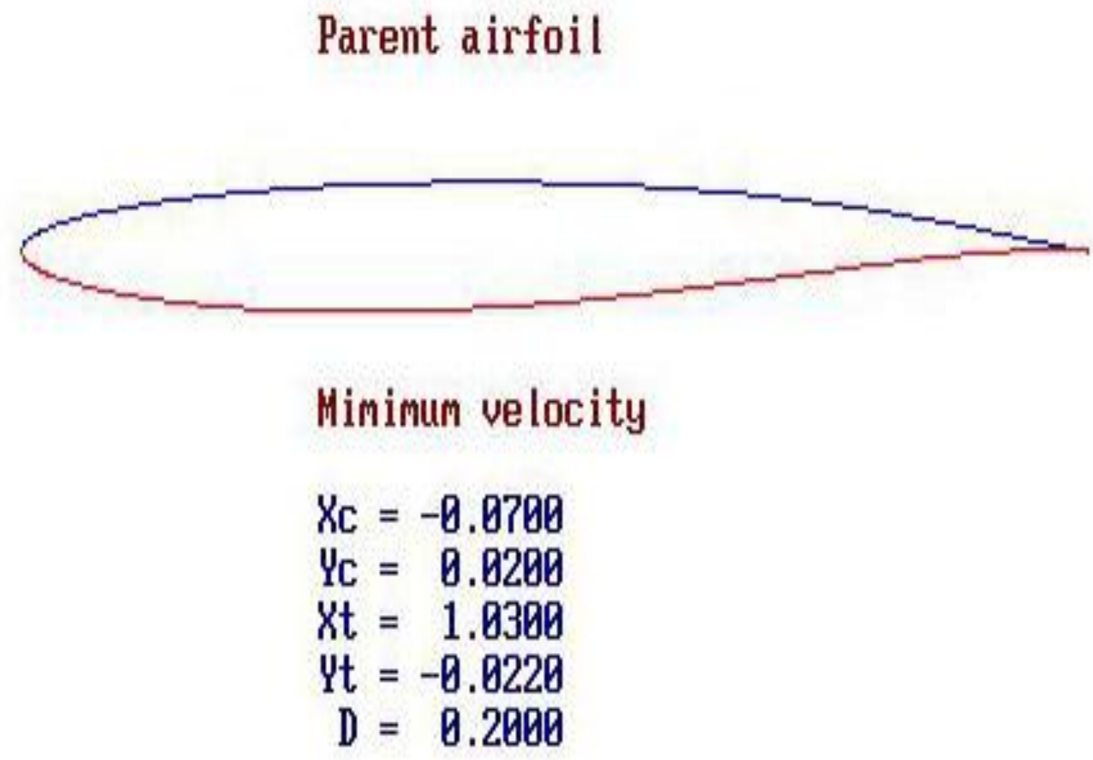

Parent airfoil
Laminar flow
Xc = -0.0707
Yc = 0.0840
Xt = 1.0400
Yt = 0.0000
D = 0.0000


Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl=-0.25
Cl=-0.04
Cl= 0.17
Cl= 0.38
Cl= 0.59
0.5
Airfoil-section: reflex
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0

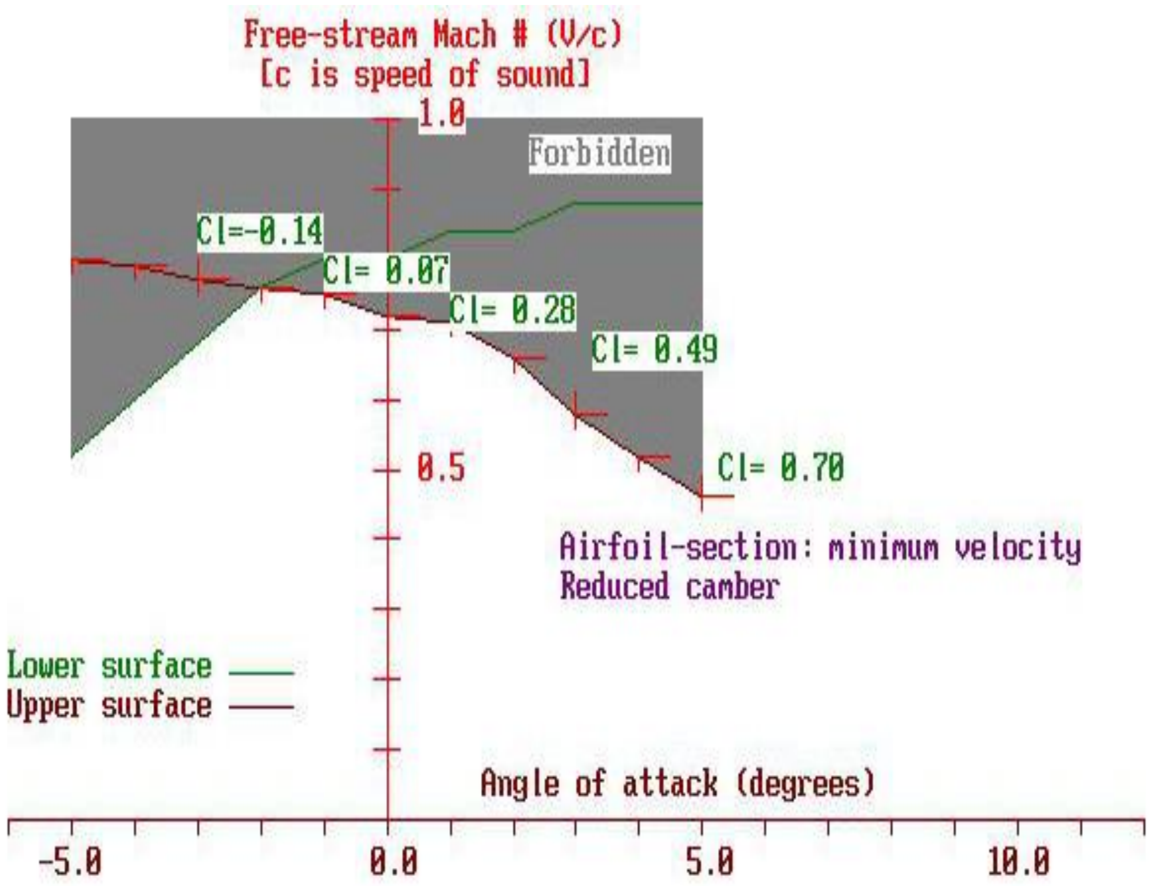

Parent airfoil
Reflex, tailess
Xc = -0.0940
Yc = 0.0340
Xt = 1.0300
Yt = 0.0220
D = 0.0000



Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl=-0.09
Cl= 0.12
Cl= 0.33
Cl= 0.54
Cl= 0.75
0.5
Airfoil-section: minimum velocity
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0



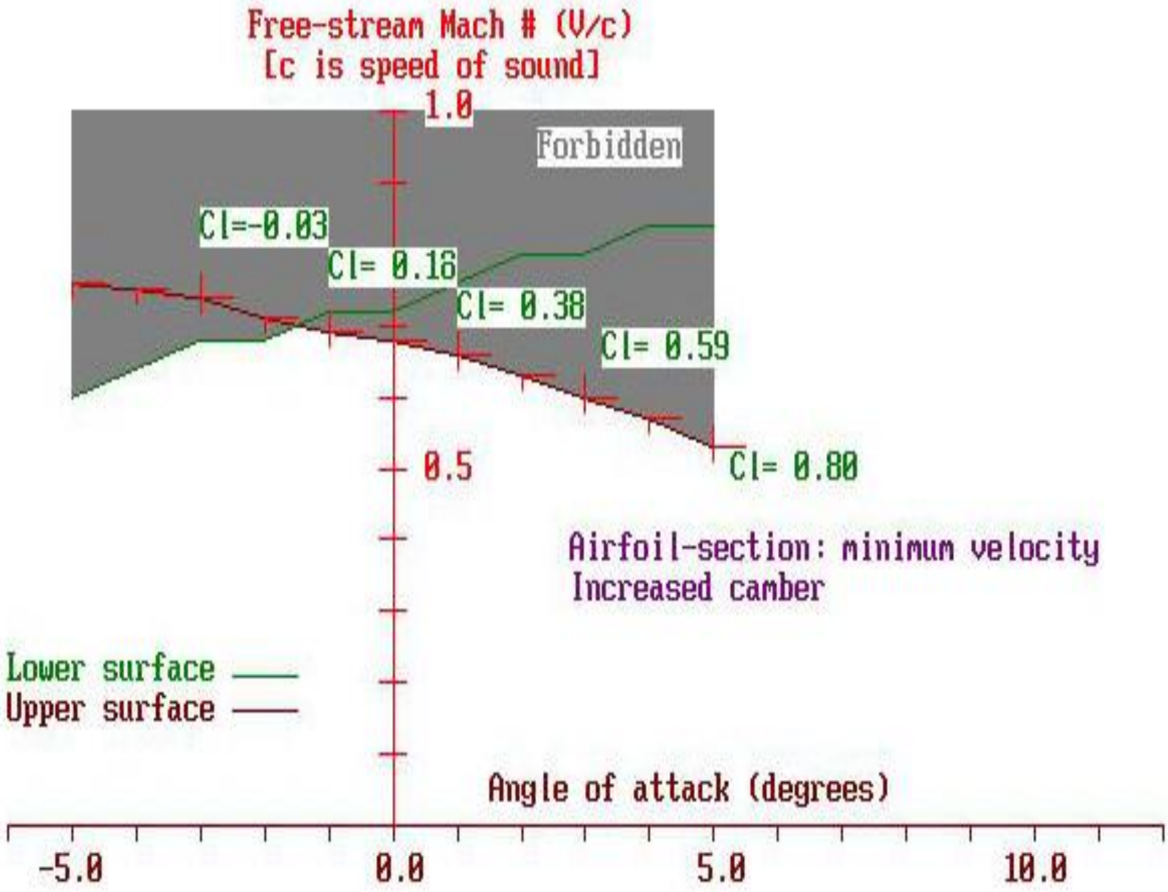
Parent airfoil
Mimimum velocity
Xc = -0.0700
Yc = 0.0200
Xt = 1.0300
Yt = -0.0220
D = 0.2000

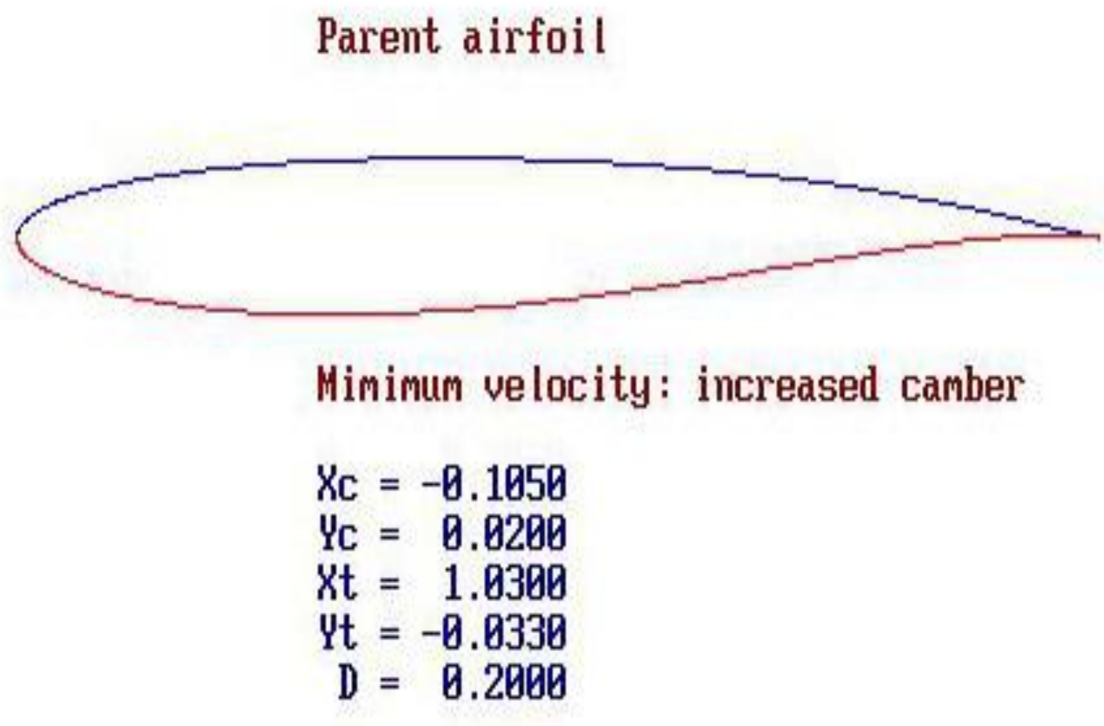


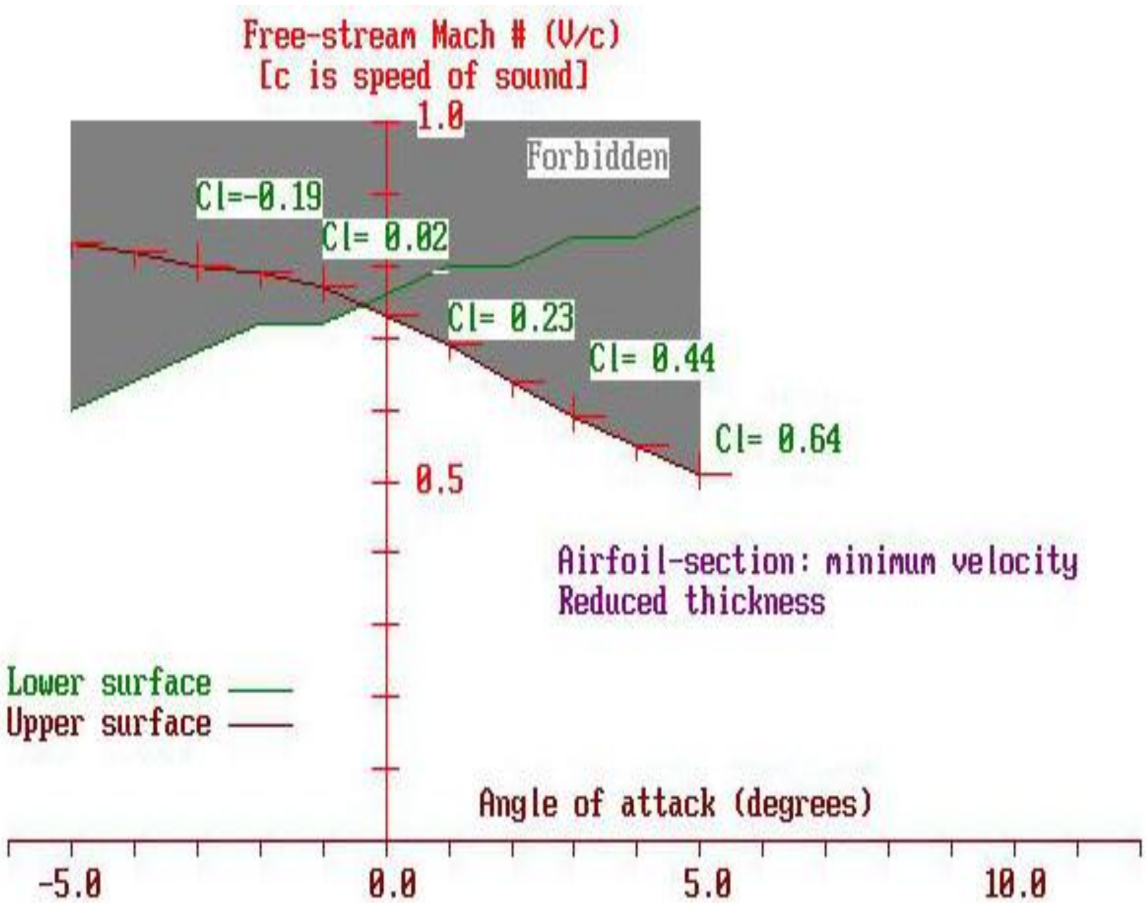
Parent airfoil

Mimimum velocity: reduced camber

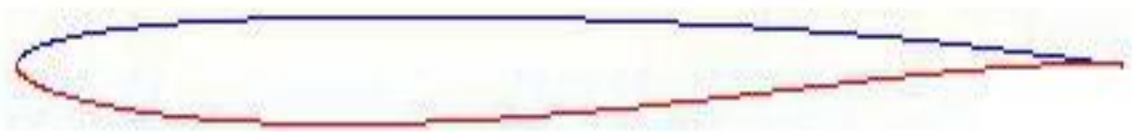
Xc = -0.0350
Yc = 0.0200
Xt = 1.0300
Yt = -0.0110
D = 0.2000


Free-stream Mach # (V/c)
[c is speed of sound]
1.0
Forbidden
Cl=-0.03
Cl= 0.16
Cl= 0.38
Cl= 0.59
Cl= 0.80
0.5
Airfoil-section: minimum velocity
Increased camber
Lower surface
Upper surface
Angle of attack (degrees)
-5.0
0.0
5.0
10.0


Parent airfoil
Mimimum velocity: increased camber
Xc = -0.1050
Yc = 0.0200
Xt = 1.0300
Yt = -0.0330
D = 0.2000

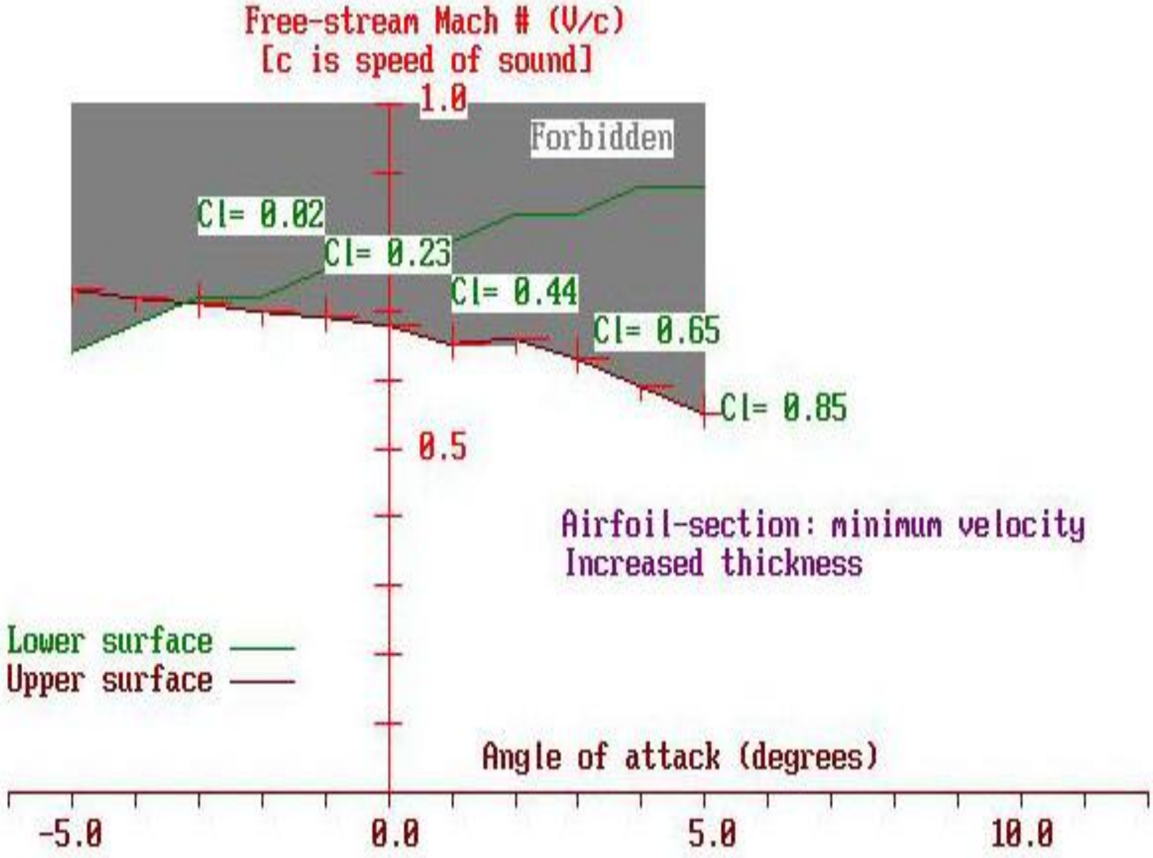


Parent airfoil

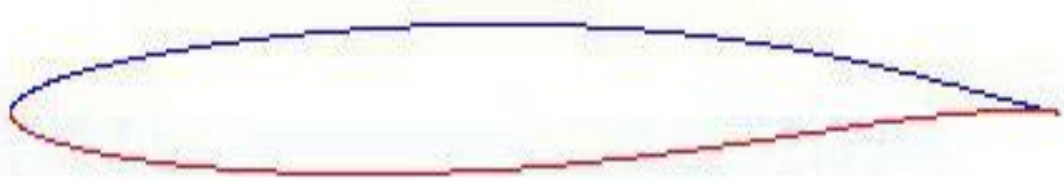


Mimimum velocity: reduced thickness

Xc = -0.0700
Yc = 0.0000
Xt = 1.0150
Yt = -0.0220
D = 0.2000



Parent airfoil



Mimimum velocity: increased thickness

Xc = -0.0700
Yc = 0.0400
Xt = 1.0450
Yt = -0.0220
D = 0.2000

Now, do we have a winner?

Scanning through the preceding airfoils, some salient points emerge.

1. The intersection of the lines marked "Lower surface" and "Upper surface" coincide with the simultaneous formation of a shock wave on both surfaces.

2. The "best airfoil" is Jones' "Minimum velocity" section. The lift coefficient available without any shock wave forming is for the free-stream Mach number of .72 at which point the lift coefficient is 0.2.

3. Geometrically speaking, the upper surface has moderate to low "upper camber", while the lower surface has fairly pronounced "lower camber". There is a small downward cusp.

4. Changing either the camber or the thickness does not improve the minimum velocity section.

5. The section is reasonably thick, suggesting that when the Prandtl-Glauert compressibility factor is applied, the tip sections will not be ridiculously thin (we hope).

6. The improvement of the minimum velocity section over Clark-Y is about 0.07 Mach units, a worthwhile gain.

7. To actually achieve the benefit of this section, the minimum velocity section will need to be very accurately formed in the manufacture of the prop.

Time to move on to consideration of section drag in Part 12.

References:

1. Robert T. Jones (1988). "Modern Subsonic Aerodynamics", Appendix A. Aircraft Designs Inc.
2. Captain Eric Brown (2012). "Miles M52. Gateway to Supersonic Flight", p162. Spellmount.
3. Charles Hampson Grant (1941). "Model Airplane Design. Theory of Flight", p11. Jay Publishing Corporation, New York.
4. P. B. Shelley . "Ozymandias of Egypt", p261. The Golden Treasury. Thomas Nelson.

Critical Mach # Part 12: **Section drag at high Mach numbers.**

The preceding eleven Parts of this discourse has been aimed at choosing a propeller airfoil which will at least produce some thrust out near the tips. The chosen cut-off point has been the critical Mach number. Effects of viscosity have been ignored in their entirety.

Some justifications of this rather extreme approach can be listed.

1. Propeller efficiency has two components: induced and profile. These components each account for about 6% of the total propeller efficiency of, say, 88%. Only the profile efficiency is affected by viscosity. If viscosity is ignored, we will get an apparent efficiency of 94%, an error of 6.5%. Not nice, but not a huge error: indeed, it is possible to design a reasonable propeller ignoring viscosity altogether.

2. The losses due to viscosity occur in a layer of air close to the propeller surface: the boundary layer. If we allow a shock wave to form, the result will be an interaction of the shock with the boundary layer which causes the flow to break away from the surface, thereby moving the streamlines behind the shock away from the surface. The region thus created consists of turbulent air, which, while creating no lift, absorbs energy from the stream. This is seen as drag. Further, the shock then rides on the top of the turbulent layer, moving rearward, further degrading the flow. This is the reason we have attempted to avoid the formation of the shock altogether.

3. The viscous nature of the air is characterised by the Reynolds number, which is the product of chord by free-stream velocity. The local nature of the flow, in terms of viscosity, is thereby ignored when using the Reynolds number. However, in the flow leading over the forward surfaces, the boundary layer is thin: as they say, there is a favourable pressure gradient, which resists flow breakaway. In the method we have described so far, only the forward part of the airfoil has analysed: it follows that our assumptions are reasonable in terms of viscous forces.

4. We have been dealing with airfoil sections as a function of Mach number. The fact is that the variation of drag with Mach number is very small: indeed, the drag falls with Mach number increase, provided no shock forms. This is consistent with an increasing "local Reynolds number", if such a thing existed. An assumption of a drag coefficient of 0.02 for all values of Mach number is reasonable.

5. With a fixed value of drag coefficient, the drag-to-lift ratio depends only on the lift coefficient: for that reason, we require the highest possible lift coefficient from our chosen airfoil at the operating angle-of-attack. Thus a value of 0.2, which we have seen in our charts for the minimum velocity airfoil, gives a useful lift-to-drag ratio of 10. This is not atypical for both supersonic jets and hang gliders!

6. Now we can fiddle the books a bit, since our critical Mach number of 0.72 is still well below Mach unity. There is a phenomenon known as "tip relief". Propellers seem always to perform better than they should. The gain is equivalent to 10% of Mach number at Mach unity. Therefore, we can design for Mach 0.9, even though the true tip speed is Mach unity. The growth in drag above Mach 0.72 is slow to start with, so our deficit of 0.18 Mach units need not be a disaster. Further, we can hope that by forming both upper and lower shocks nearly simultaneously, the

strength of the combined shock is less than, say, a single strong shock from Clark-Y or similar airfoil.

So much for creative accounting for shock waves: this concludes the 12 part discussion of propeller airfoil selection for high Mach numbers. Cheers.

Appendix A1: Program to generate surface velocities on an airfoil.

```
 '  AIRFOIL.BAS

 '  Oshkosh airfoil program     QUICK.BASIC        14/10/2012

      '  Adapted from the Oshkosh Airfoil Program, 1984, first written by
Rick
      '  McWilliams and further adapted by Thomas Gans.

      '  Incompressible flow calculation by Jones/Joukowski transform.

      '  Data for various angles of attack, velocities/pressures
      '  along the streamlines from surface only.

      '  Surface pair only. Put into angle of attack/free-stream velocity
loops.

      '  Each pair of streamlines stored on one file. The paired
      '  streamlines are close together to permit velocity calculation.

      '  Chose airfoil surface as dividing line.
      '  Get surface velocity distribution using flow continuity
      '  between streamlines and potential derivative.

      '  Also returns  Cl, Cm, ACl0

      ' Data: Clark-Y  from Jones "Subsonic Aerodynamics".
      ' Supercool/Joukowski airfoil calculation.
      '-------------------------------------------------------------------

      ' Airfoil parameters  (with value ranges)

      '    AoA = angle of attack (-10 to +10 degrees)
      '      D = thickness parameter , delta (-1.0, to 1.0)
      '     XC = forward thickness parameter (-.4 to 0.0)
      '     XT = middle thickness parameter  (1.0 to 1.4)
      '     YC = middle camber parameter     (-0.4 to 0.4)
      '     YT = aft camber (reflex) parameter (-0.4 to 0.4)
      '------------------------------------------------------------------

          DIM Xair(1000), Yair(1000), camberx(500), cambery(500)
          DIM x(2000), y(2000)
          DIM xstream(4, 2000), ystream(4, 2000)
          DIM svelocity(2, 2000), station(2, 2000), pressure(2, 2000)
          DIM ifile$(20)

      '  Rounding functions, conversions

          DEF FNR (x) = INT(100 * x + .5) / 100
          DEF FNR2 (x) = INT(x * 10000 + .5) / 10000

          pi = 4 * ATN(1): dtr = pi / 180: rtd = 180 / pi
          drive$ = "C:\CRITICAL\"        ' save streamlines here

          SCREEN 9, , 1, 1: CLS
      '-----------------------------------------------------------------

      ' Choose airfoil parameters for Jones/Joukowsky calculation

    ' Clark Y
       'Xc = -.08: Yc = .085: Xt = 1.02: Yt = .017: D = 0 ' Clark-Y
       'section$ = "Clark-Y"
```

```
   ' Clark -ve camber
      'Xc = -.08: Yc = -.085: Xt = 1.02: Yt = -.017: D = 0 ' Clark-Y
      'section$ = "Clark-Y negative camber"

   ' zero-camber Clark-Y
      ' Xc = -.08: Yc = 0!: Xt = 1.02: Yt = -.01: D = 0   ' Clark-Y zero
camber
      ' section$ = "Clark-Y  zero camber"

   ' Laminar flow airfoil
        'Xc = -.0707: Yc = .084: Xt = 1.04: Yt = 0: D = 0    ' Laminar
        'section$ = "Laminar flow"

   ' Reflex airfoil
        'Xc = -.094: Yc = .034: Xt = 1.03: Yt = .022: D = 0    ' Reflex
        'section$ = "Reflex, tailess"

   ' Minimum velocity flow airfoil
          Xc = -.07: Yc = .02: Xt = 1.03: Yt = -.022: D = .2    ' Minimum
velocity
          section$ = "Minimum velocity"

   ' Minimum velocity, reduced camber  airfoil
        'Xc = -.035: Yc = .02: Xt = 1.03: Yt = -.011: D = .2    ' Minimum
velocity
        ' section$ = "Minimum velocity: reduced camber"

   ' Minimum velocity, increased camber airfoil
         'Xc = -.105: Yc = .02: Xt = 1.03: Yt = -.033: D = .2    ' Minimum
velocity
         'section$ = "Minimum velocity: reduced camber"

   ' Minimum velocity, reduced  thickness flow airfoil
         'Xc = -.07: Yc = 0: Xt = 1.015: Yt = -.022: D = .2    ' Minimum
velocity
         'section$ = "Mimimum velocity: reduced thickness"

   ' Minimum velocity, increased  thickness flow airfoil
         'Xc = -.07: Yc = .04: Xt = 1.045: Yt = -.022: D = .2     ' Minimum
velocity
         'section$ = "Mimimum velocity: increased thickness camber"

     ' Lancair IV
         ' xc = -.086: yc = .03: Xt = 1.051: Yt = .004: D = .1
         ' section$="Lancair IV"

     ' Initialise airfoil parameters and variables

     ' AoA loop  ... main program

             ifile = 0
           FOR AoA = -5 TO 5.05 STEP 1   ' angle-of-attack (degrees)
             ifile = ifile + 1
             ifile$ = STR$(ifile)
             GOSUB AoAchange         ' change angle-of-attack
             GOSUB ploop            ' derive/show airflow/pressure/velocity
           NEXT AoA

     ' save AoA counter

        OPEN drive$ + "counter.dat" FOR OUTPUT AS #1
          PRINT #1, ifile
        CLOSE #1
```

```
           PRINT ifile
              INPUT "End main program .. streamlines saved", vbv: CLS
              END

    '----------------------------------------------------------------

     ' Establish loops to generate and store the streamlines and
     ' velocity/pressure distributions for a range of angles-of-attack

        ' Program branch to display airflow over airfoil
        ' Airfoil surface is the flow dividing line

ploop: :
           WINDOW (-5, -4)-(5, 4): CLS : LINE (-5, -2)-(5, 3), 14, BF

        '----------------------------------------------------------

              GOSUB xdraw           ' draw airfoil at current angle of
attack

        ' Save airfoil coordinates

           OPEN drive$ + "air" + ifile$ + ".dat" FOR OUTPUT AS #1
                    PRINT #1, nc, LEx
                  FOR i = 1 TO nc
                    PRINT #1, Xair(i), Yair(i)
                  NEXT i
           CLOSE #1

     ' Derive stream-lines

              istart = 0
              istop = 0
              istep = .02

          OPEN drive$ + "loopdata.dat" FOR OUTPUT AS #1
              PRINT #1, istart, istop, istep
          CLOSE #1

          FOR ks = istart TO istop STEP istep      ' each ks is a
streamline
                                                   ' ie stream function
value
               ifile$ = STR$(ifile)

               IF ks < 0 THEN del = -1 ELSE del = 1

      ' Upper streamlines

          jj = 1: n(jj) = 0: COLOR 12                   '  K>0, upper
surface
        k = ks + .007 * del: GOSUB drawstreamlines      '  +ve dividing
line
                                                        '  when ks=0
          jj = 2: n(jj) = 0: COLOR 12
        k = ks + .01 * del: GOSUB drawstreamlines       ' very close
streamline

      ' Plot upper streamlines

            jj = 1
        FOR kk = 2 TO n(jj)
            LINE (xstream(jj, kk - 1), ystream(jj, kk - 1))-(xstream(jj,
kk), ystream(jj, kk)), 2
```

```
         NEXT kk

             jj = 2
         FOR kk = 2 TO n(jj)
            LINE (xstream(jj, kk - 1), ystream(jj, kk - 1))-(xstream(jj,
kk), ystream(jj, kk)), 4
         NEXT kk

      ' Now derive velocity and pressure curves for upper streamlines

               jj = 2
             duma = (ystream(jj - 1, 1) - ystream(jj, 1)) ^ 2
             dumb = (xstream(jj - 1, 1) - xstream(jj, 1)) ^ 2
           vwidth = SQR(duma + dumb)                     ' free=stream
velocity

               po = 3                    ' starting point for lower
intercept search
               counter = 0

           FOR mm = 4 TO n(jj) - 1 STEP 1     ' run along all points from
TE forward

              ' drop normal from outer streamline (jj = 2)

                   xw = xstream(jj, mm): yw = ystream(jj, mm) '      test
point
                    x = xw: y = yw
                       dums1 = ystream(jj, mm) - ystream(jj, mm + 1)
                       dums2 = xstream(jj, mm) - xstream(jj, mm + 1)
                   s2 = -1 / (dums1 - dums2)
                   b2 = ystream(jj, mm) - s2 * xstream(2, mm)

              ' intercept with inner line

                 FOR p = po TO n(jj)

                  x = xstream(jj - 1, p): y = ystream(jj - 1, p): COLOR 5
                 s1 = (ystream(jj - 1, p) - ystream(jj - 1, p - 1)) /
(xstream(1, p) - xstream(1, p - 1))
                 b1 = ystream(jj - 1, p - 1) - s1 * xstream(jj - 1, p - 1)

                  xs = -(b2 - b1) / (s2 - s1) ' lower stream line
                  ys = s1 * xs + b1           ' intercept to upper normal

                IF xs >= xstream(jj - 1, p) THEN GOTO ww

              NEXT p
ww:
          LINE (xw, yw)-(xs, ys) ' lower stream line intercept to upper
normal
                                 ' reset initial loop value to current
value
              po = p: h = 1
             counter = counter + 1
             swidth = SQR((xw - xs) ^ 2 + (yw - ys) ^ 2)
             svelocity(h, counter) = vwidth / swidth
             station(h, counter) = xw
             pressure(h, counter) = 1 - svelocity(h, counter) ^ 2

           NEXT mm                                    ' move along
streamline

      ' Plot stream velocity versus chord
```

```
           mj(1) = counter
          FOR ii = 2 TO mj(1)
           LINE (station(h, ii - 1), svelocity(h, ii - 1))-(station(h, ii),
svelocity(h, ii)), 11
           LINE (station(h, ii - 1), pressure(h, ii - 1))-(station(h, ii),
pressure(h, ii)), 10
          NEXT ii

abc:
          jj = 3: n(jj) = 0: COLOR 5                        ' K<0, lower
surface
          k = -ks - .005 * del: GOSUB drawstreamlines   ' -ve dividing line

          jj = 4: n(jj) = 0: COLOR 5
          k = -ks - .02 * del: GOSUB drawstreamlines

      ' Derive and plot lower streamlines

             jj = 3
         FOR kk = 2 TO n(jj)
            LINE (xstream(jj, kk - 1), ystream(jj, kk - 1))-(xstream(jj,
kk), ystream(jj, kk)), 2
         NEXT kk

             jj = 4
         FOR kk = 2 TO n(jj)
            LINE (xstream(jj, kk - 1), ystream(jj, kk - 1))-(xstream(jj,
kk), ystream(jj, kk)), 4
         NEXT kk

      ' Now derive velocity and pressure curves for lower streamlines

               jj = 4
             duma = (ystream(jj - 1, 1) - ystream(jj, 1)) ^ 2
             dumb = (xstream(jj - 1, 1) - xstream(jj, 1)) ^ 2
           vwidth = SQR(duma + dumb)                      ' free=stream
velocity

               po = 3: h = 2                 ' starting point for lower
intercept search
               counter = 0

           FOR mm = 4 TO n(jj) - 1 STEP 1     ' run along all points from
TE forward

              ' drop normal from outer streamline (jj = 4)

                   xw = xstream(jj, mm): yw = ystream(jj, mm) '      test
point
                   x = xw: y = yw

                       dums1 = ystream(jj, mm) - ystream(jj, mm + 1)
                       dums2 = xstream(jj, mm) - xstream(jj, mm + 1)
                   s2 = -1 / (dums1 - dums2)
                   b2 = ystream(jj, mm) - s2 * xstream(jj, mm)

              ' intercept with inner line

                 FOR p = po TO n(jj)

                     x = xstream(jj - 1, p): y = ystream(jj - 1, p): COLOR
5
```

```
                s1 = (ystream(jj - 1, p) - ystream(jj - 1, p - 1)) /
(xstream(jj - 1, p) - xstream(jj - 1, p - 1))
                b1 = ystream(jj - 1, p - 1) - s1 * xstream(jj - 1, p - 1)

                   xs = -(b2 - b1) / (s2 - s1)    ' lower stream line
                   ys = s1 * xs + b1              ' intercept to upper
normal

               IF xs >= xstream(jj - 1, p) THEN GOTO www

             NEXT p
www:
          LINE (xw, yw)-(xs, ys)   ' lower stream line intercept to upper
normal
             po = p               ' reset initial loop value to current
value
               counter = counter + 1

               swidth = SQR((xw - xs) ^ 2 + (yw - ys) ^ 2)
               svelocity(h, counter) = vwidth / swidth
               station(h, counter) = xw
               pressure(h, counter) = 1 - svelocity(h, counter) ^ 2

           NEXT mm                                  ' move along
streamline

      ' Plot stream velocity versus chord

             mj(2) = counter
          FOR ii = 2 TO mj(2)
           LINE (station(h, ii - 1), svelocity(h, ii - 1))-(station(h, ii),
svelocity(h, ii)), 12
           LINE (station(h, ii - 1), pressure(h, ii - 1))-(station(h, ii),
pressure(h, ii)), 9
          NEXT ii

          LINE (LEx, -.1)-(LEx, .3), 5           ' mark leading edge x-
abciss

     '---------------------------------------------------------------------

     ' save files

        OPEN drive$ + "strea" + ifile$ + ".dat" FOR OUTPUT AS #1

                 PRINT #1, mj(1), mj(2), ifile$
                 PRINT #1, Cl, Cm, ACl0, AoA

           FOR jj = 1 TO 3.0005 STEP 2            ' streamline closest to
ks
                 PRINT #1, n(jj)
               FOR n = 1 TO n(jj)
                 PRINT #1, xstream(jj, n), ystream(jj, n)
               NEXT n

               FOR hh = 1 TO 2                     ' data closest to ks
                 FOR ii = 1 TO mj(hh)              ' derived data
                   PRINT #1, station(hh, ii), svelocity(hh, ii),
pressure(hh, ii)
                 NEXT ii
               NEXT hh
           NEXT jj

        CLOSE #1
```

```
        NEXT ks                          ' next stream function

        RETURN

airfoiltransform:              ' Subroutine for airfoil transformation

        XB = CA * x - SA * y: YB = SA * x + CA * y
        x0 = A * XB + Xc: Y0 = A * YB + Yc
        xd = x0 - D
        D2 = xd * xd + Y0 * Y0
        XP = x0 - (EX * xd + EY * Y0) / D2
        YP = Y0 - (EY * xd - EX * Y0) / D2
        R2 = XP * XP + YP * YP
        xa = XP + XP / R2                ' offset
        ya = YP - YP / R2
        xr = CA * xa + SA * ya           ' rotation
        yr = -SA * xa + CA * ya
      RETURN

streamcorrection:                    ' Subroutine for stream corrections

        M = x ^ 2 + y ^ 2
        p = y - y / M + G / 2 * LOG(M)
       DP = 1 + (2 * y ^ 2 - M) / (M ^ 2) + G * y / M
        y = y + (k - p) / (DP + .01)
      RETURN



drawstreamlines:                     ' Subroutine to draw streamlines

          x = -50
          y = (k + SGN(k) * SQR(k ^ 2 + 4)) / 2
2525        GOSUB streamcorrection
            IF ABS(k - p) > .01 THEN 2525
           first = 0

2540      x = -x
          GOSUB airfoiltransform
          x1 = S3: y1 = S4        ' initial values of S3 and S4 are zero
          S3 = xr
          S4 = yr
          x2 = S3: y2 = S4
          LINE (x2, y2)-(x2, y2), 4

          n(jj) = n(jj) + 1:
          xstream(jj, n(jj)) = x2
          ystream(jj, n(jj)) = y2

2615      x = -x
           M = x * x + y * y
           u = (1 - (x * x - y * y) / (M * M)) + G * y / M
           v = -2 * x * y / (M * M) - G * x / M

           IF first = 0 THEN DX = 50 - 1.7 ELSE DX = .005
           first = 1

           DY = DX + v / u
            IF ABS(DY) > .005 THEN DY = SGN(DY) * .005

          x = x + DX
          y = y + DY
```

```
2660    GOSUB streamcorrection
        IF ABS(k - p) > .01 THEN 2660
         IF x < 2 THEN GOTO 2540
         x0 = x: VV0 = VV
        RETURN               ' Return for next stream-line

AoAchange:
         logo = 1
         EX = (Xt - D) * (Xt - 1) - Yt ^ 2
         EY = Yt * (Xt - D + Xt - 1)
         A = SQR((Xt - Xc) ^ 2 + (Yc - Yt) ^ 2)

      ' Calculate airfoil geometry + Cl  Cm  ACl0 for current AoA

430       P0 = 1: Z1 = 0: Z2 = 0: Z3 = 0: Z4 = 0: Z5 = 0: Z6 = 0
          SA = SIN(AoA * dtr): CA = COS(AoA * dtr)
          EX = (Xt - D) * (Xt - 1) - Yt ^ 2: EY = Yt * (Xt - D + Xt - 1)
          A = SQR((Xt - Xc) ^ 2 + (Yc - Yt) ^ 2): AA = A ^ 2 - Xc ^ 2 - Yc
^ 2

      ' Calculate lift and moment of airfoil

          G = 2 * (SA * (Xt - Xc) - CA * (Yt - Yc)) / A
          Cl = 3 * G                               ' lift coeff
          Cm = -1.4 * Yc + 2.21 * Yt               ' pitching moment coeff
          ACl0 = -57.3 * (Yc - Yt) / (Xt - Xc)     ' zero lift angle
          RETURN

xdraw:
        ' Draw the airfoil

          scl = 5: WINDOW (-.7 * scl, -.6 * scl)-(.8 * scl, .6 * scl)

            jj = 0
        FOR th = .01 TO 2 * pi STEP .01     ' th is angle theta around
circle
             x = COS(th)
             y = SIN(th)
           GOSUB airfoiltransform
            jj = jj + 1
             Xair(jj) = xr
             Yair(jj) = yr
             x1 = x2: y1 = y2
             x2 = xr: y2 = yr
           IF th = .01 THEN 2205

           IF x2 < 0 AND x2 < x1 THEN LEx = x1       ' leading edge x-
abcissa
2205        NEXT th
            nc = jj
        RETURN
```

Appendix A2: Program to compute and display critical Mach number.

```
' CHART.BAS                                              14/10/2012
' Reference text: Supersonic Aerodynamics" Edward R. C. Miles 1950.
' Oshkosh/Joukowsky data plots.

' Loads data from AIRFOIL.BAS, rescales to (0,0) (1,0) and plots
' velocities against M*, both upper and lower surfaces.
' Forbidden region due to initiation of shocks shown.

' Gets true pressure, local mach number

 DIM Xair(1000), Yair(1000), camberx(500), cambery(500)
 DIM x(2000), Y(2000)
 DIM xstream(4, 2000), ystream(4, 2000)
 DIM svelocity(2, 2000), station(2, 2000), pressure(2, 2000)
 DIM ifile$(20), clocal(2, 2000), Mstarlocal(2, 2000)
 DIM Mcrit(2, 120), McritX(2, 120), Freevel(120)
 DIM AoA(50), ACL0(50), Cl(50), Freevelu(50), Freevell(50)

        CLEAR , , 4096         ' Enlarge stack
        SCREEN 9, , 1, 1
        PALETTE 0, 55          ' true White background

       pi = 4 * ATN(1): dtr = pi / 180: rtd = 180 / pi
       drive$ = "C:\CRITICAL\"        ' save streamlines here
       scl = 1.2: WINDOW (-.3 * scl, -1 * scl)-(1.3 * scl, 3 *
scl): CLS

    '------------------------------------------------------------------

    ' Fixed data for getting critical Mfree number location

      k = 1.4                             ' cp/cv   or 5/7
      co = 340                            ' speed of sound m/s at M = 0

      Vmax = SQR(2 * co ^ 2 / (k - 1))    ' maximum value for adiabatic
flow
                                            ' eq (1-17) p13 with
co^2=k(P/rho)
                                            ' from eq (1-24) p17
      cstar = SQR((k - 1) / (k + 1)) * Vmax  ' critical value of c, for V =
c

         ifile = 1: ifile$ = STR$(ifile)
       GOSUB plotfoil                        ' rescale and draw airfoil
         ifile = 0: ifile$ = STR$(ifile)

     ' Load AoA counter from AIRFOIL.BAS

        OPEN drive$ + "counter.dat" FOR INPUT AS #1
          INPUT #1, AoAfile
        CLOSE #1

             nfree = 0

                kk = 0
          FOR AoAx = 1 TO AoAfile
                kk = kk + 1
```

```
             GOSUB loaddata

        ' upper surface, AoA Vs Mfree
        ' Mfree is free-stream Mach number

                startfree = .45: stopfree = .9: stepfree = .01

           FOR Mfree = startfree TO stopfree STEP stepfree
               GOSUB pressure
                  hh = 1                          ' upper surface
               GOSUB crit
               GOSUB mcritplot
                IF Mcrit(hh, kk) < .1 THEN GOTO tl ELSE GOTO ty3
tl:        NEXT Mfree
ty3:          Freevelu(kk) = Mfree

          ' lower surface, AoA Vs Mfree

                startfree = 0!: stopfree = .9: stepfree = .04
           FOR Mfree = startfree TO stopfree STEP stepfree
              GOSUB pressure
                  hh = 2                          ' lower surface
              GOSUB crit
               IF Mcrit(hh, kk) < .01 THEN GOTO tl1 ELSE GOTO ty2
tl1:       NEXT Mfree
ty2:              Freevell(kk) = Mfree
                  IF Mcrit(hh, kk) = 0 THEN Freevell(kk) = 0

         NEXT AoAx

     '------------------------------------------------------------------

        ' Print/plot data

             CLS
             LOCATE 3, 1: COLOR 14: PRINT "    Angle of attack / critical
point"

             PRINT : COLOR 3: PRINT "   Upper surface (suction side)"
             PRINT : COLOR 12
             PRINT "   kk    AoA   ACl0     Cl    Freevelocity     Mcrit
location  "
             PRINT

           FOR n = 1 TO kk
             COLOR 4: PRINT USING " ###   ##.##  ##.##   ##.##"; n; AoA(n);
ACL0(n); Cl(n);
             COLOR 4: IF Freevelu(n) < .001 THEN COLOR 0
              PRINT USING "           #.##     ###.##    ###.##";
Freevelu(n); Mcrit(2, n); McritX(2, n)
           NEXT n

             COLOR 3: PRINT : PRINT "   Lower surface (pressure side)"
             PRINT : COLOR 12
             PRINT "   kk    AoA   ACL0     Cl    Freevelocity     Mcrit
location"
             PRINT

           FOR n = 1 TO kk
             COLOR 2: PRINT USING " ###   ##.##  ##.##   ##.##"; n; AoA(n);
ACL0(n); Cl(n);
             COLOR 2: IF Freevell(n) < .001 THEN COLOR 0
              PRINT USING "           #.##     ###.##    ###.##";
Freevell(n); Mcrit(2, n); McritX(2, n)
```

```
          NEXT n

      INPUT vbv

       ' Plot Mcrit / Freestream Vs AoA


            WINDOW (-7, -.2)-(13, 1.2): CLS

            ' shade forbidden values of free-stream

                 FOR n = 1 TO kk - 1
                   LINE (AoA(n), Freevelu(n))-(AoA(n + 1), 1), 8, BF
                 NEXT n

          ' Forbidden region upper surface

                FOR n = 2 TO kk
                     Ann = AoA(n - 1): Amm = AoA(n)
                     xFnn = Freevelu(n - 1): Fmm = Freevelu(n)
                   FOR A = Ann TO Amm STEP .01
                     F = (Fmm - xFnn) * (A - Ann) / (Amm - Ann) + xFnn
                     LINE (A, F)-(Amm, F), 8
                   NEXT A
                NEXT n

          ' Forbidden region lower surface

                FOR n = 1 TO kk - 2
                     Ann = AoA(n): Amm = AoA(n + 1)
                     xFnn = Freevell(n): Fmm = Freevell(n + 1)
                        IF Fmm <= .001 THEN GOTO hg
                   FOR A = Ann TO Amm STEP .01
                     F = (Fmm - xFnn) * (A - Ann) / (Amm - Ann) + xFnn
                     LINE (A, F)-(AoA(1), F), 8
                   NEXT A
hg:             NEXT n

                    LOCATE 5, 38: COLOR 8: PRINT "Forbidden"

                    LOCATE 18, 5: COLOR 2: PRINT "Lower surface"
                       LINE (-2.5, .21)-(-1.5, .21), 2
                    LOCATE 19, 5: COLOR 4: PRINT "Upper surface"
                       LINE (-2.5, .16)-(-1.5, .16), 4

              LOCATE 15, 40: COLOR 5: PRINT "Airfoil-section: minimum
velocity"

                LINE (0, 0)-(0, 1), 12: LOCATE 4, 20
                FOR i = .1 TO 1.001 STEP .1: LINE (-.2, i)-(.2, i), 12:
NEXT i
                  LOCATE 4, 31: COLOR 12: PRINT "1.0"
                  LOCATE 13, 31: PRINT "0.5"

                LINE (-6, 0)-(12, 0), 6: LOCATE 21, 35: COLOR 6: PRINT
"Angle of attack (degrees)"
                FOR i = -6 TO 12 STEP 1: LINE (i, -.02)-(i, 0): NEXT i
                     LOCATE 23, 7: PRINT "-5.0"
                     LOCATE 23, 28: PRINT "0.0"
                     LOCATE 23, 48: PRINT "5.0"
                     LOCATE 23, 67: PRINT "10.0"

                LOCATE 2, 20: COLOR 12: PRINT "Free-stream Mach # (V/c)"
                LOCATE 3, 20: PRINT " [c is speed of sound]"
```

```
          FOR n = 2 TO kk
               IF Mcrit(1, n) < .0001 THEN GOTO mmm
                 LINE (AoA(n - 1), Freevelu(n - 1))-(AoA(n), Freevelu(n)),
4
mmm:           IF Mcrit(2, n) < .0001 THEN GOTO nnn
                  LINE (AoA(n - 1), Freevell(n - 1))-(AoA(n), Freevell(n)),
2
nnn:      NEXT n

                 FOR n = 1 TO kk
                   LINE (AoA(n), Freevelu(n))-(AoA(n) + .5, Freevelu(n))
                   LINE (AoA(n), Freevelu(n))-(AoA(n), Freevelu(n) - .02)
                 NEXT n

                 FOR n = 3 TO kk STEP 2
                   LINE (AoA(n), Freevelu(n))-(AoA(n) + .5, Freevelu(n))
                   LINE (AoA(n), Freevelu(n))-(AoA(n), Freevelu(n) + .03),
12
                 NEXT n

                   COLOR 2
                   LOCATE 7, 17: PRINT USING "Cl=##.##"; Cl(3)
                   LOCATE 8, 25: PRINT USING "Cl=##.##"; Cl(5)
                   LOCATE 9, 33: PRINT USING "Cl=##.##"; Cl(7)
                   LOCATE 10, 42: PRINT USING "Cl=##.##"; Cl(9)
                   LOCATE 12, 50: PRINT USING "Cl=##.##"; Cl(11)

         COLOR 0: INPUT vbv
        END

loaddata:

     ' Load  streamlines/pressure/velocity

          OPEN drive$ + "loopdata.dat" FOR INPUT AS #1
              INPUT #1, istart, istop, istep
          CLOSE #1

            getx = istart + 20 * istep
            getx = istart

      FOR ks = getx TO istop STEP istep
            ifile = ifile + 1
            ifile$ = STR$(ifile)

         OPEN drive$ + "strea" + ifile$ + ".dat" FOR INPUT AS #1

                  INPUT #1, mj(1), mj(2), ifile$
                  INPUT #1, Cl, Cm, ACL0, AoA
                  AoA(kk) = AoA: ACL0(kk) = ACL0: Cl(kk) = Cl
                  nfile = VAL(ifile$)

           FOR jj = 1 TO 3.0005 STEP 2             ' streamline closest to
ks
                  INPUT #1, n(jj)
                FOR n = 1 TO n(jj)
                  INPUT #1, xstream(jj, n), ystream(jj, n)
                NEXT n

        ' rescale streamline for plotting

            FOR n = 1 TO n(jj)
              xstream(jj, n) = (xstream(jj, n) - LEx) / dum
```

```
            NEXT n

       ' load velocity and pressure data

            FOR hh = 1 TO 2                      ' data closest to ks
              FOR ii = 1 TO mj(hh)               ' derived data
                INPUT #1, station(hh, ii), svelocity(hh, ii), pressure(hh,
ii)
                station(hh, ii) = (station(hh, ii) - LEx) / dum ' rescale
              NEXT ii
            NEXT hh
         RETURN

pressure:
       ' Calculate pressure based on St. Venant / Wentzel eq.
       ' Assume velocities unchanged by the effect of compressibility (P-G
rule)
       ' Also get local speed of sound at all stations

         FOR hh = 1 TO 2                         ' data closest to ks
           FOR ii = 1 TO mj(hh)                  ' derived data

         'eq. (1-31) p18
            Mstar = svelocity(hh, ii) * co * Mfree / cstar' Mfree is free
stream mach#

         'eq. (1-35) p19
            pressure(hh, ii) = (1 - (k - 1) / (k + 1) * Mstar ^ 2) ^ (k /
(k - 1))

         'eq. (1-25) p17
            clocal(hh, ii) = SQR((k - 1) / 2 * (Vmax ^ 2 - (Mfree * co *
svelocity(hh, ii)) ^ 2))
            Mstarlocal(hh, ii) = svelocity(hh, ii) * co * Mfree /
clocal(hh, ii)

          NEXT ii
        NEXT hh

      NEXT jj
        CLOSE #1

             GOSUB dataplot

        NEXT ks                         ' load next file of streamline data
      RETURN


dataplot:
           ' Plot data for streamlines and velocities as rescaled

      ' Plot remote point on streamline

         FOR jj = 1 TO 3.0001 STEP 2
           LINE (xstream(jj, 1), ystream(jj, 1))-(xstream(jj, 1),
ystream(jj, 1)), 2
         NEXT jj

      ' Plot dividing streamline

            IF ks = getx THEN GOTO sad ELSE GOTO happy
sad:
        ' upper dividing line
```

```
            FOR n = 2 TO n(1) - 2
              dumm = svelocity(1, n) * Mfree
              COLOR 14: IF dumm > cstar / co THEN COLOR 4
              LINE (xstream(1, n - 1), ystream(1, n - 1))-(xstream(1, n),
ystream(1, n))
            NEXT n

        ' lower dividing line

            FOR n = 2 TO n(3) - 2
             LINE (xstream(3, n - 1), ystream(3, n - 1))-(xstream(3, n),
ystream(3, n)), 2
            NEXT n
happy:

      ' Plot streamline

          FOR jj = 1 TO 1.0001 STEP 2              ' upper streamline
            FOR n = 2 TO n(jj) - 2

                dum2 = Mstarlocal(1, n)
                dum1 = Mfree * svelocity(1, n)      ' normalise to free-
stream

                 IF dum1 > cstar / co THEN COLOR VAL(A$) ELSE GOTO rr
                 IF dum2 < 1.153 AND dum2 > 1.147 THEN COLOR 13
                 IF dum2 < 1.103 AND dum2 > 1.097 THEN COLOR 13
                 IF dum2 < 1.053 AND dum2 > 1.047 THEN COLOR 13
                 IF dum2 < 1.003 AND dum2 > .997 THEN COLOR 13
                 IF dum2 < .953 AND dum2 > .9475 THEN COLOR 13

              LINE (xstream(jj, n - 1), ystream(jj, n - 1))-(xstream(jj,
n), ystream(jj, n))
rr:         NEXT n
          NEXT jj

          FOR jj = 3 TO 3.0001 STEP 2             ' lower streamline
            FOR n = 2 TO n(jj) - 2
                dum1 = Mfree * svelocity(2, n)
                IF dum1 > cstar / co THEN COLOR VAL(A$) ELSE GOTO tt
              LINE (xstream(jj, n - 1), ystream(jj, n - 1))-(xstream(jj,
n), ystream(jj, n))
tt:         NEXT n
          NEXT jj

' stored critical data
                 Flag1 = 0
                 Flag2 = 0

          FOR hh = 1 TO 2                     ' data closest to ks

                  IF hh = 1 THEN COLOR 4      ' upper surface
                  IF hh = 2 THEN COLOR 14     ' lower surface

                  IF hh = 1 THEN COLOR 2      ' upper surface
                  IF hh = 2 THEN COLOR 9     ' lower surface

                  IF hh = 1 THEN COLOR 4      ' upper surface
                  IF hh = 2 THEN COLOR 2     ' lower surface

        NEXT hh
      RETURN

crit:
```

```
      ' Find and store critical Mach number, station, free-stream velocity

                Mcrit(hh, kk) = 0
                McritX(hh, kk) = 0

                IF hh = 1 THEN jjx = 1                ' upper surface
                IF hh = 2 THEN jjx = 3                ' lower surface

          FOR ii = 130 TO mj(hh) - 100

           IF Mstarlocal(hh, ii) > cstar / co THEN GOTO ww ELSE GOTO qq
ww:           Mcrit(hh, kk) = Mstarlocal(hh, ii)      ' trap critcal mach #
              McritX(hh, kk) = station(hh, ii)
              GOTO trs
qq:       NEXT ii

trs:        IF Mcrit(hh, kk) <> 0 THEN GOTO pp ELSE GOTO rrs
pp:          LOCATE 2, 10: COLOR 4: PRINT USING "Free stream Mach number:
#.##"; Mfree
             LOCATE 4, 12: COLOR 9
            IF hh = 1 THEN PRINT " Mcritical     x/c "
            IF hh = 1 THEN LOCATE 6, 14: COLOR 12
            IF hh = 1 THEN PRINT USING "#.###      #.###   Upper surface";
Mcrit(hh, kk); McritX(hh, kk)

            IF hh = 2 THEN PRINT " Mcritical     x/c "
            IF hh = 2 THEN LOCATE 7, 14
            IF hh = 2 THEN COLOR 2
            IF hh = 2 THEN PRINT USING "#.###      #.###   Lower surface";
Mcrit(hh, kk); McritX(hh, kk)

rrs:        RETURN


plotfoil:    'load and rescale airfoil to cords (0,0) at LE and (1,0) at TE

              OPEN drive$ + "air" + ifile$ + ".dat" FOR INPUT AS #1
                   INPUT #1, nc, LEx
                  FOR i = 1 TO nc
                    INPUT #1, Xair(i), Yair(i)
                  NEXT i
              CLOSE #1


              FOR i = 1 TO nc STEP 1
                 Xair(i) = Xair(i) - LEx     ' shift to LE origin (0,0)
              NEXT i

                 dum = Xair(nc)
              FOR i = 1 TO nc STEP 1
                 Xair(i) = Xair(i) / dum     'scale to TE at (1,0)
              NEXT i

           RETURN

mcritplot:
           ' Plot streamline up to Mcrit

          FOR jj = 1 TO 1.0001 STEP 2             ' upper streamline
            FOR n = 2 TO n(jj) - 2

                dum2 = Mstarlocal(1, n)
                dum1 = Mfree * svelocity(1, n)      ' normalise to free-
stream
```

```
                 IF dum1 > cstar / co THEN COLOR VAL(A$) ELSE GOTO rr1
                 IF dum2 < 1.153 AND dum2 > 1.147 THEN COLOR 13
                 IF dum2 < 1.103 AND dum2 > 1.097 THEN COLOR 13
                 IF dum2 < 1.053 AND dum2 > 1.047 THEN COLOR 13
                 IF dum2 < 1.003 AND dum2 > .997 THEN COLOR 13
                 IF dum2 < .953 AND dum2 > .9475 THEN COLOR 13

              LINE (xstream(jj, n - 1), ystream(jj, n - 1))-(xstream(jj,
n), ystream(jj, n))
rr1:         NEXT n
          NEXT jj

          FOR jj = 3 TO 3.0001 STEP 2             ' lower streamline
            FOR n = 2 TO n(jj) - 2
                dum1 = Mfree * svelocity(2, n)
                IF dum1 > cstar / co THEN COLOR VAL(A$) ELSE GOTO tt1
              LINE (xstream(jj, n - 1), ystream(jj, n - 1))-(xstream(jj,
n), ystream(jj, n))
tt1:         NEXT n
          NEXT jj

               RETURN
```