Compressibility: Selection of propeller airfoil section. (Prandtl-Glauert rule)

Previously, we have noted "Tilley's Law"; which states, the thinner the section at the propeller tip, the faster the airplane. There exists another rule, the Prandtl-Glauert rule, which says much the same thing, but throws in a little formula as well. I must admit, it has taken me the best part of 20 years to figure out how then to use this P-G rule, while Tilley's law is almost universally applied.

The idea behind the P-G rule is that we cannot use the same airfoil at 270 m/s (M = 0.8) as we might use at 30 m/s (M = 0.1). Since propeller blades have this wide variation of airspeed along their length, we have a problem. The cause of the problem relates to the behaviour of air. As speed rises, the flow of air over the airfoil becomes progressively changed, the air in the close vicinity of the airfoil surface becoming increasingly dense. This is the phenomenon of "compressibility".

Suppose we have a nice airfoil section like NACA 4412. That is, 4% camber, camber high-point at 40% of chord and thickness-to-chord ratio 12%. This section is very like Clark Y, which is a famous section of known good performance at low speed. We would like to use 4412 at high speed on a propeller, say at Mach 0.7. Can we do this, given the large change in density of the air at this speed?

Well, according to Prandtl and Glauert, we can, provided we apply a "stretch" to the airfoil in the direction of flight. A stretch? Let's see what that means on some diagrams.

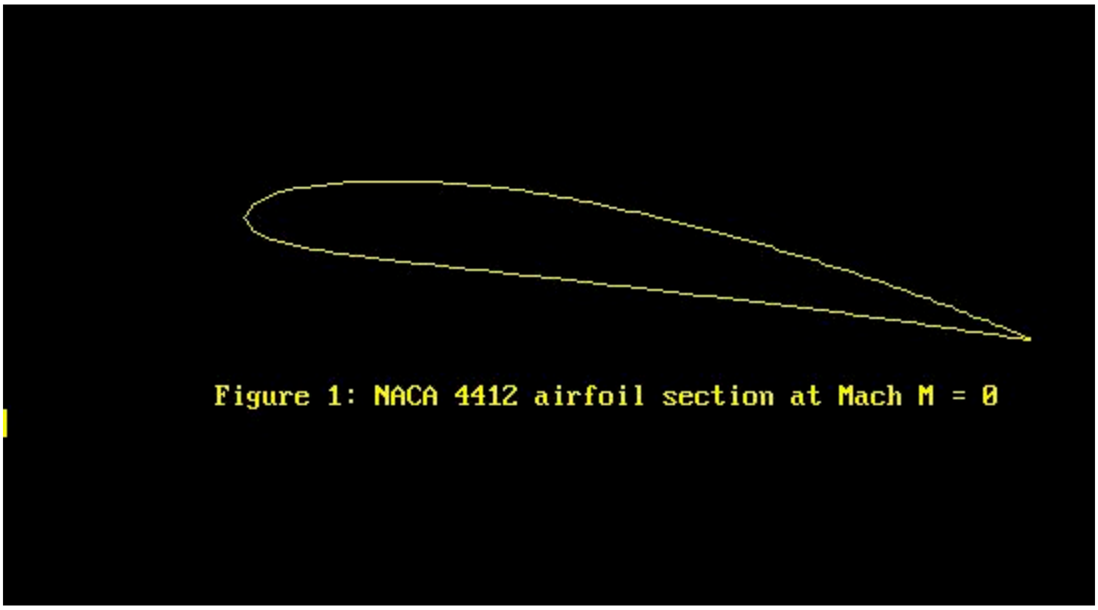


Figure 1 above is just to show what NACA 4412 looks like: it is very much like Clark Y.

The airfoil in the diagram is set to an angle of attack (measured to chord line) of 10 degrees. A more normal value would be 4 degrees. As the effect of compressibility is to lower the required angle of attack as Mach number rises, I have chosen the larger value purely for illustrative reasons.

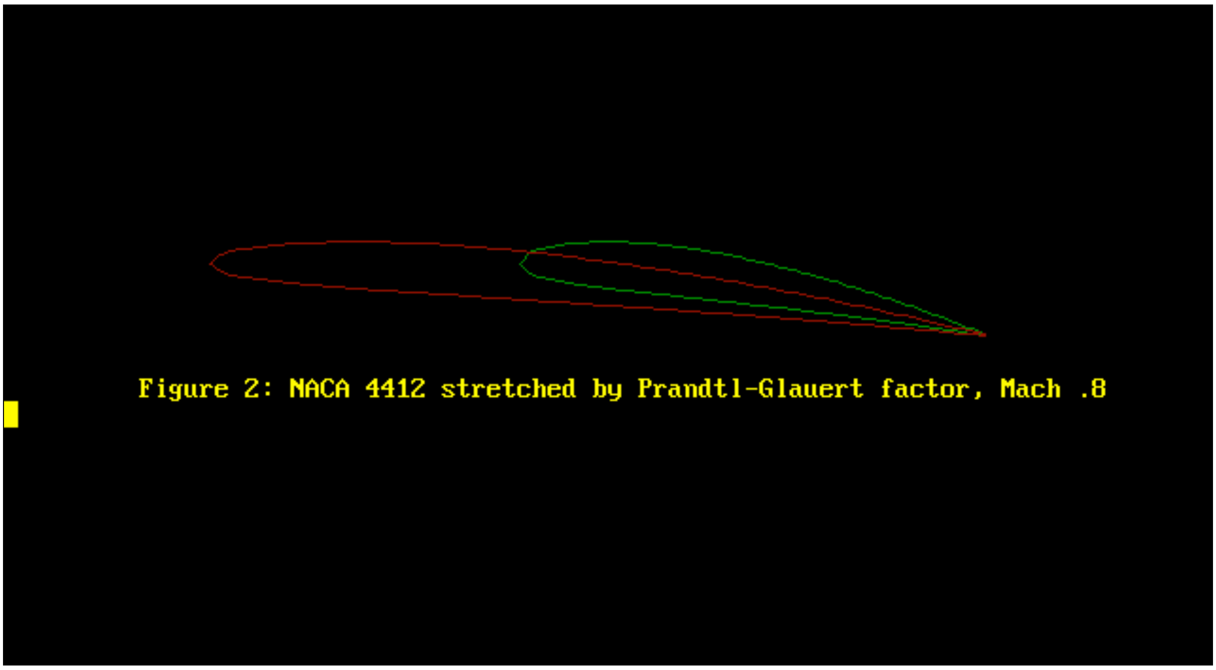
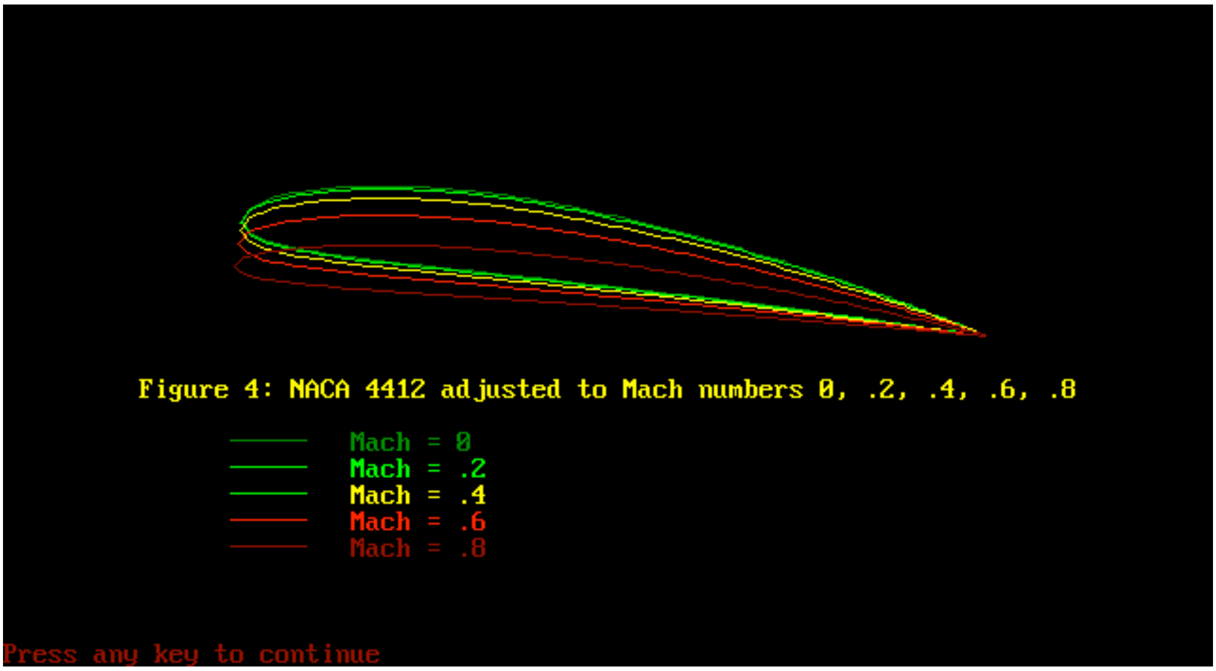


Figure 2 above shows NACA 4412 again, at Mach 0 (M = 0) but this time with the stretched version of the same airfoil at M = 0.8. This second airfoil is obtained by multiplying all the x-coordinate (abscissa) values by the Prandtl-Glauert factor k, where k is given below:

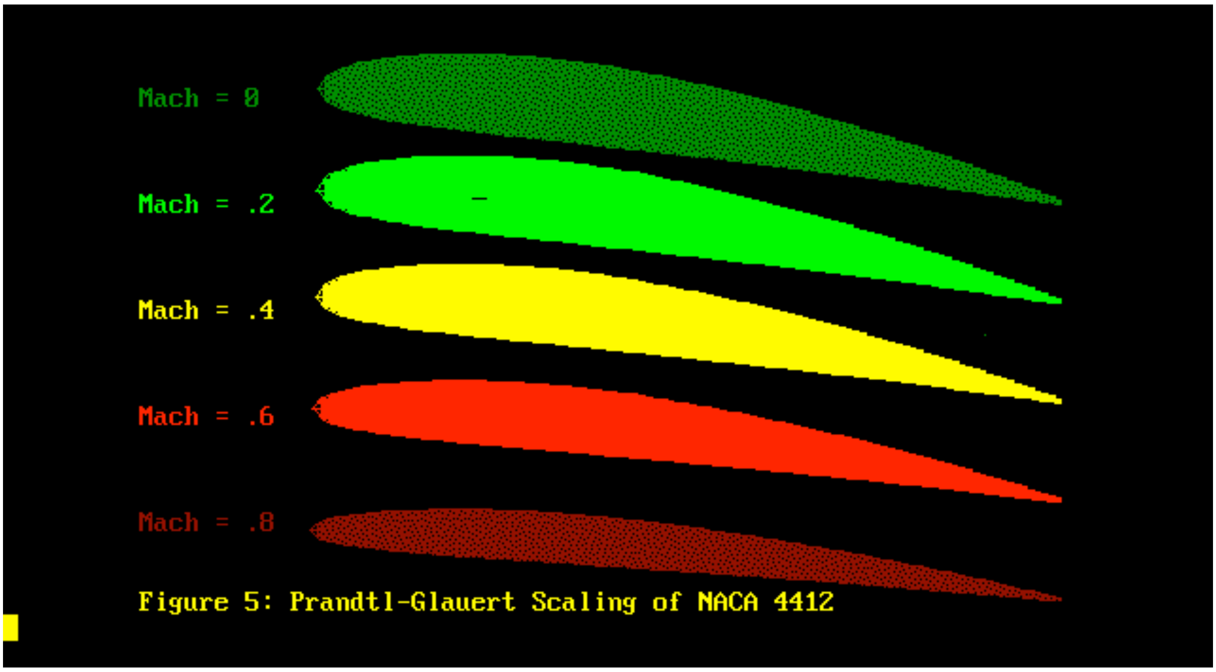
k = 1/SQR(1 – M^2)

Note that k is always greater than unity, getting bigger as M, the Mach number, increases. Figure 2 shows that the angle of attack has decreased, and the chord increased. This increase in chord is not required, we can easily scale it back to the original chord. This is done in Figure 3 below, where all 3 figures are now superimposed.

Returning to our propeller airfoil design problem, recall we wanted to use an airfoil like NACA 4412 all the way along the prop. If we choose stations along the prop corresponding to speeds of M = 0.2, 0.4, 0.6 and 0.8, then we can use the Prandtl-Glauert rule to find what NACA 4412 would like that with the compressibility taken into account. This exercise is displayed in Figure 4, again with all the airfoils superimposed.



Note that the effect of the rule has been to decrease angle of attack, reduce camber and to reduce thickness ratio as Mach number increases. The tip airfoil has been thinned, as one would hope for consistency with Tilley's law. The sections are displayed again in Figure 5, but this time they are not overlaid.



By scaling the airfoil sections in this manner, we have attempted to retain the good properties of NACA 4412. In particular, we have held the amount of lift generated by the section to be the same at every station, irrespective of the nature of compressible flow at the chosen Mach number.

To recap, as Mach number increases, we must reduce angle of attack, camber and thickness ratio to maintain a fixed value of lift coefficient.

Note that I quit at Mach 0.8. In full size airplane wings, the rule is generally not used above Mach 0.7. But propellers are a little kinder than wings, and we can stretch the rule a little. Above Mach 0.8, a new phenomenon appears, related to compressibility but not obedient to the Prandtl-Glauert rule. This new phenomenon we have seen before, namely, the formation of shock waves as the local flow over the airfoil section starts to exceed M =1.0.

The good properties of NACA 4412 simply cannot be retained at local velocities for $M > 1$ It We then require the "supercritical" sections, which, quite unlike NACA 4412, have negative camber and cusped trailing edges.